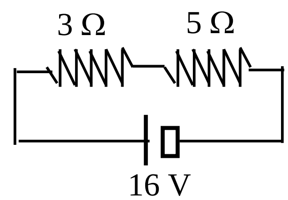
Chapter 28. Direct-Current Circuits

Resistors in Series and Parallel (Ignore internal resistances for batteries in this section.)

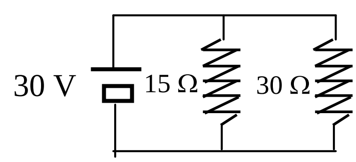
28-1. A 5-Ω resistor is connected in series with a 3-Ω resistor and a 16-V battery. What is the effective resistance and what is the current in the circuit?



$R_e = R_1 + R_2 = 3\ \Omega + 5\ \Omega$; $\boxed{R_e = 8.00\ \Omega}$

$$I = \frac{V}{R} = \frac{16\ \text{V}}{8\ \Omega} \qquad \boxed{I = 2.00\ \text{A}}$$

28-2. A 15-Ω resistor is connected in parallel with a 30-Ω resistor and a 30-V source of emf. What is the effective resistance and what total current is delivered?



$$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{(15\ \Omega)(30\ \Omega)}{15\ \Omega + 30\ \Omega}; \qquad \boxed{R_e = 10.0\ \Omega}$$

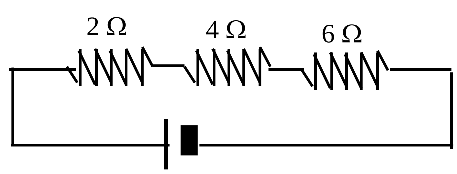
$$I = \frac{V}{R} = \frac{30\ \text{V}}{10\ \Omega}; \qquad \boxed{I = 3.00\ \text{A}}$$

28-3. In Problem 28-2, what is the current in 15 and 30-Ω resistors?

For Parallel: $V_{15} = V_{30} = 30\ V$; $I_{15} = \dfrac{30\ \text{V}}{15\ \Omega}$; $\boxed{I_{15} = 2.00\ \text{A}}$

$I_{30} = \dfrac{30\ \text{V}}{30\ \Omega}$; $\boxed{I_{30} = 1.00\ \text{A}}$ Note: $I_{15} + I_{30} = I_T = 3.00$ A

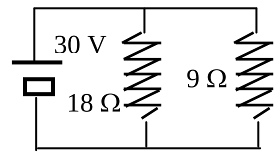
28-4. What is the equivalent resistance of 2, 4, and 6-Ω resistors connected in parallel?



$R_e = 2\ \Omega + 4\ \Omega + 6\ \Omega$; $\boxed{R_e = 12.0\ \Omega}$

28-5. An 18-Ω resistor and a 9-Ω resistor are first connected in parallel and then in series with a 24-V battery. What is the effective resistance for each connection? Neglecting internal resistance, what is the total current delivered by the battery in each case?

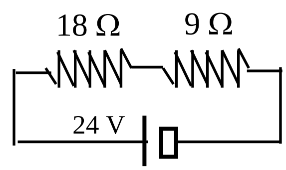
$$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{(18\ \Omega)(9\ \Omega)}{18\ \Omega + 9\ \Omega}; \qquad \boxed{R_e = 6.00\ \Omega}$$



$$I = \frac{V}{R} = \frac{24\ \text{V}}{6.00\ \Omega}; \qquad \boxed{I = 4.00\ \text{A}}$$

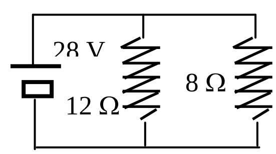
$$R_e = R_1 + R_2 = 18\ \Omega + 9\ \Omega; \qquad \boxed{R_e = 27.0\ \Omega}$$

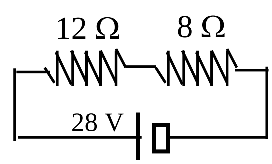
$$I = \frac{V}{R} = \frac{24\ \text{V}}{27\ \Omega} \qquad \boxed{I = 0.889\ \text{A}}$$



28-6. A 12-Ω resistor and an 8-Ω resistor are first connected in parallel and then in series with a 28-V source of emf. What is the effective resistance and total current in each case?

$$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{(12\ \Omega)(8\ \Omega)}{12\ \Omega + 8\ \Omega}; \qquad \boxed{R_e = 4.80\ \Omega}$$





$$I = \frac{V}{R} = \frac{28\ \text{V}}{4.80\ \Omega}; \qquad \boxed{I = 5.83\ \text{A}}$$

$$R_e = R_1 + R_2 = 12\ \Omega + 8\ \Omega; \qquad \boxed{R_e = 20.0\ \Omega} \qquad I = \frac{V}{R} = \frac{28\ \text{V}}{20\ \Omega} \qquad \boxed{I = 1.40\ \text{A}}$$

28-7. An 8-Ω resistor and a 3-Ω resistor are first connected in parallel and then in series with a 12-V source. Find the effective resistance and total current for each connection?

$$R_e = \frac{(3\ \Omega)(8\ \Omega)}{3\ \Omega + 8\ \Omega}; \qquad \boxed{R_e = 2.18\ \Omega} \qquad I = \frac{V}{R} = \frac{12\ \text{V}}{2.18\ \Omega}; \qquad \boxed{I = 5.50\ \text{A}}$$

$$R_e = R_1 + R_2 = 3\ \Omega + 8\ \Omega; \qquad \boxed{R_e = 11.0\ \Omega} \qquad I = \frac{V}{R} = \frac{12\ \text{V}}{11\ \Omega} \qquad \boxed{I = 1.09\ \text{A}}$$

28-8. Given three resistors of 80, 60, and 40 Ω, find their effective resistance when connected in series and when connected in parallel.

Series: $R_e = 80\ \Omega + 60\ \Omega + 40\ \Omega$; $\boxed{R_e = 180\ \Omega}$

Parallel: $\frac{1}{R_e} = \sum\frac{1}{R_i} = \frac{1}{80\ \Omega} + \frac{1}{60\ \Omega} + \frac{1}{40\ \Omega}$; $\boxed{R_e = 18.5\ \Omega}$

28-9. Three resistances of 4, 9, and 11 Ω are connected first in series and then in parallel. Find the effective resistance for each connection.

Series: $R_e = 4\ \Omega + 9\ \Omega + 11\ \Omega$; $\boxed{R_e = 24.0\ \Omega}$

Parallel: $\frac{1}{R_e} = \sum\frac{1}{R_i} = \frac{1}{4\ \Omega} + \frac{1}{9\ \Omega} + \frac{1}{11\ \Omega}$; $\boxed{R_e = 2.21\ \Omega}$

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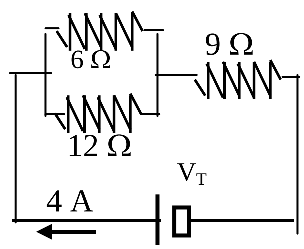
*28-10. A 9-Ω resistor is connected in series with two parallel resistors of 6 and 12 Ω. What is the terminal potential difference if the total current from the battery is 4 A?

$R_e = \frac{(6\ \Omega)(12\ \Omega)}{6\ \Omega + 12\ \Omega} = 4\ \Omega$; $R_e = 4\ \Omega + 9\ \Omega = 13\ \Omega$

$V_T = IR = (4\text{ A})(13\ \Omega)$; $\boxed{V_T = 52.0\text{ V}}$



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*28-11. For the circuit described in Problem 28-10, what is the voltage across the 9-Ω resistor and what is the current through the 6-Ω resistor?

$V_9 = (4\text{ A})(9\ \Omega) = 36\text{ V}$; $\boxed{V_9 = 36.0\text{ V}}$

The rest of the 52 V *drops across each of the parallel resistors:*

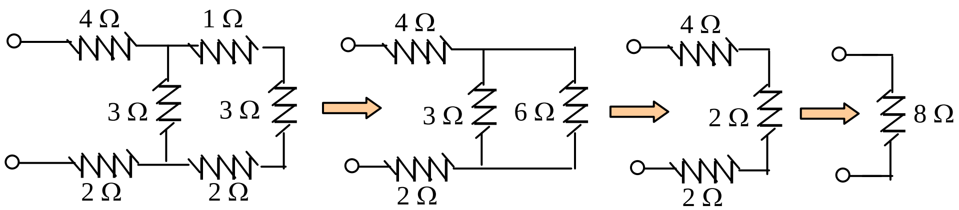
$V_6 = V_7 = 52\text{ V} - 36\text{ V}$; $V_6 = 16\text{ V}$

$I_6 = \frac{V_6}{R_6} = \frac{16\text{ V}}{6\ \Omega}$; $\boxed{I_6 = 2.67\text{ A}}$

*****28-12. Find the equivalent resistance of the circuit drawn in Fig. 28-19.

Start at far right and reduce circuit in steps: $R' = 1\ \Omega + 3\ \Omega + 2\ \Omega = 6\ \Omega$;

$$R'' = \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 2\ \Omega;\quad R_e = 2\ \Omega + 4\ \Omega + 2\ \Omega;\quad \boxed{R_e = 8\ \Omega}$$

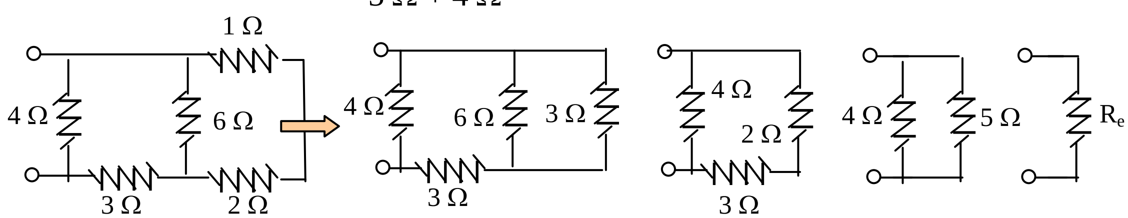


*****28-13. Find the equivalent resistance of the circuit shown in Fig. 28-20.

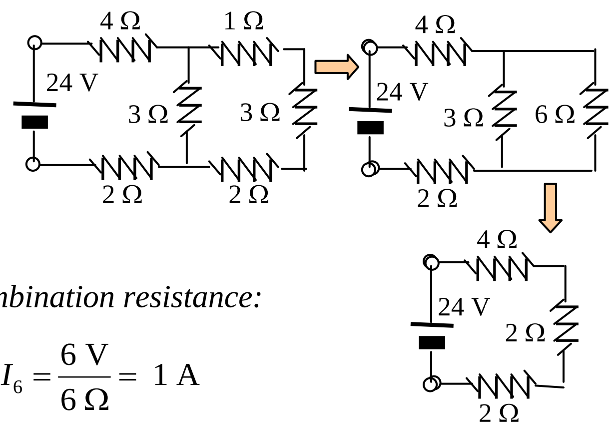
Start at far right and reduce circuit in steps: $R = 1\ \Omega + 2\ \Omega = 3\ \Omega$;

$$R' = \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 2\ \Omega;\quad R'' = 2\ \Omega + 3\ \Omega = 5\ \Omega$$

$$R_e = \frac{(5\ \Omega)(4\ \Omega)}{5\ \Omega + 4\ \Omega} = 2.22\ \Omega;\quad \boxed{R_e = 2.22\ \Omega}$$



*28-14. If a potential difference of 24 V is applied to the circuit drawn in Fig. 28-19, what is the current and voltage across the 1-Ω resistor?



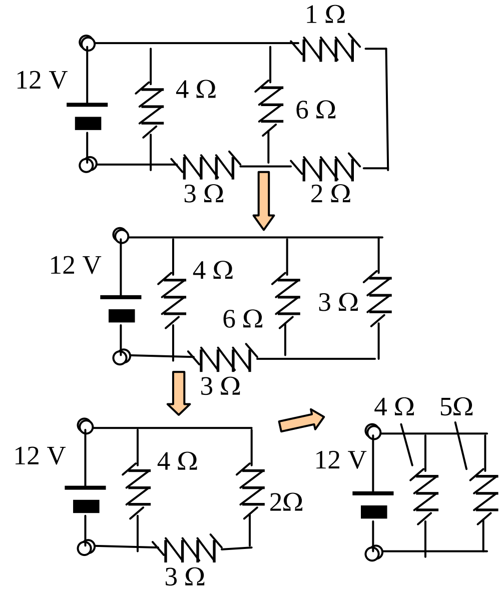
$R_e = 8.00\ \Omega$; $\quad I = \frac{24\ \text{V}}{8\ \Omega} = 3.00$ A;

The voltage across the 3 *and* 6-Ω *parallel connection is found from* I_t *and the* 2-Ω *combination resistance:*

$V_3 = V_6 = (2\ \Omega)(3.00\ \text{A})$; $\quad V_6 = 6.00$ V; $\quad I_6 = \frac{6\ \text{V}}{6\ \Omega} = 1$ A

Thus, $I_1 = I_6 = 1.00$ A, *and* $V_1 = (1\ \text{A})(1\ \Omega) = 1$ V; $\quad \boxed{V_1 = 1\ \text{V};\quad I_1 = 1\ \text{A}}$

*28-15. If a potential difference of 12-V is applied to the free ends in Fig. 28-20, what is the current and voltage across the 2-Ω resistor?



$R_e = 2.22\ \Omega$; $I = \dfrac{12\text{ V}}{2.22\ \Omega} = 5.40\text{ A}$;

Note that $V_5 = 12$ V; $I_5 = \dfrac{12\text{ V}}{5\ \Omega} = 2.40\text{ A}$

$V_{3,6} = (2.4\text{ A})(2\ \Omega) = 4.80$ V; $I_3 = \dfrac{4.8\text{ V}}{3\ \Omega} = 1.6\text{ A}$

$I_2 = I_1 = 1.60$ A; $V_2 = (1.6\text{ A})(2\ \Omega) = 3.20$ V

$I_2 = 1.60$ A; $V_2 = 3.20$ V

EMF and Terminal Potential Difference

28-16. A load resistance of 8 Ω is connected in series with a 18-V battery whose internal resistance is 1.0 Ω. What current is delivered and what is the terminal voltage?

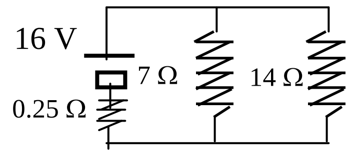
$$I = \frac{\mathcal{E}}{r + R_L} = \frac{18\text{ V}}{1.0\ \Omega + 8\ \Omega}; \quad I = 2.00\text{ A}$$

28-17. A resistance of 6 Ω is placed across a 12-V battery whose internal resistance is 0.3 Ω. What is the current delivered to the circuit? What is the terminal potential difference?

$$I = \frac{\mathcal{E}}{r + R_L} = \frac{12\text{ V}}{0.3\ \Omega + 6\ \Omega}; \quad I = 1.90\text{ A}$$

$$V_T = \mathcal{E} - Ir = 12\text{ V} - (1.90\text{ A})(0.3\ \Omega); \quad V_T = 11.4\text{ V}$$

28-18. Two resistors of 7 and 14 Ω are connected in parallel with a 16-V battery whose internal resistance is 0.25 Ω. What is the terminal potential difference and the current in delivered to the circuit?



$$R' = \frac{(7\ \Omega)(14\ \Omega)}{7\ \Omega + 14\ \Omega} = 4.67\ \Omega; \quad R_e = 0.25\ \Omega + 4.67\ \Omega$$

$$I = \frac{\mathcal{E}}{r + R'} = \frac{16\ \text{V}}{4.917\ \Omega}; \quad I = 3.25\ \text{A} \quad V_T = \mathcal{E} - Ir = 16\ \text{V} - (3.25\ \text{A})(0.25\ \Omega);$$

$$\boxed{V_T = 15.2\ \text{V}; \quad \text{I} = 3.25\ \text{A}}$$

28-19. The open-circuit potential difference of a battery is 6 V. The current delivered to a 4-Ω resistor is 1.40 A. What is the internal resistance?

$$\mathcal{E} = \text{IR}_\text{L} + \text{Ir}; \quad \text{Ir} = \mathcal{E} - \text{IR}_\text{L}$$

$$r = \frac{\mathcal{E} - IR_L}{I} = \frac{6\ \text{V - (1.40 A)(4}\ \Omega)}{1.40\ \text{A}}; \quad \boxed{r = 0.286\ \Omega}$$

28-20. A dc motor draws 20 A from a 120-V dc line. If the internal resistance is 0.2 Ω, what is the terminal voltage of the motor?

$$V_T = \mathcal{E} - Ir = 120\ \text{V} - (20\text{A})(0.2\ \Omega); \quad \boxed{V_T = 116\ \text{V}}$$

28-21. For the motor in Problem 28-21, what is the electric power drawn from the line? What portion of this power is dissipated because of heat losses? What power is delivered by the motor?

$$P_i = \mathcal{E}I = (120\ \text{V})(20\ \text{A}); \quad \boxed{P_i = 2400\ \text{W}}$$

$$P_L = I^2 r = (20\ \text{A})^2(0.2); \quad \boxed{P_L = 80\ \text{W}}$$

$$P_o = V_T I = (116\ \text{V})(20\ \text{A}); \quad \boxed{P_o = 2320\ \text{W};}$$

Note: $P_i = P_L + P_o$; 2400 W = 80 W + 2320 W

28-22. A 2-Ω and a 6-Ω resistor are connected in series with a 24-V battery of internal resistance 0.5 Ω. What is the terminal voltage and the power lost to internal resistance?

$$R_e = 2\ \Omega + 6\ \Omega + 0.5\ \Omega = 8.50\ \Omega; \quad I = \frac{\mathcal{E}}{R_e} = \frac{24\text{ V}}{8.5\ \Omega} = 2.82\text{ A}$$

$$V_T = \mathcal{E} - Ir = 24\text{ V} - (2.82\text{ A})(0.5\ \Omega); \quad \boxed{V_T = 22.6\text{ V}}$$

$$P_L = I^2 r = (2.82\text{ A})^2(0.5\ \Omega); \quad \boxed{P_L = 3.99\text{ W}}$$

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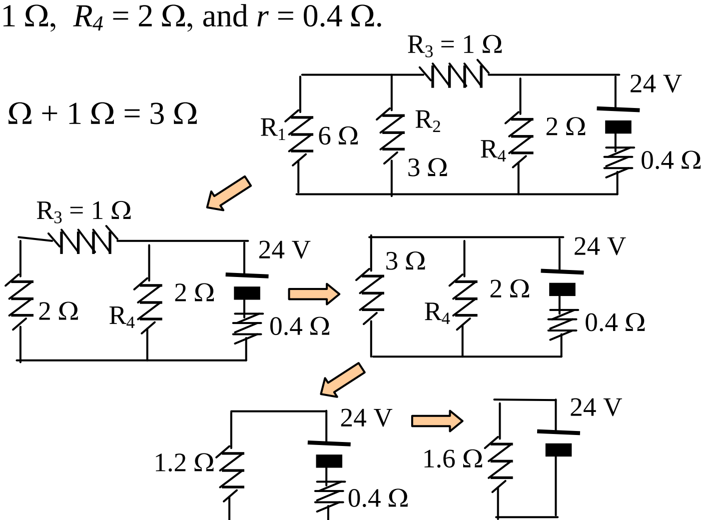
*28-23. Determine the total current and the current through each resistor for Fig. 28-21 when $\mathcal{E} = 24$ V, $R_1 = 6\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 2\ \Omega$, and $r = 0.4\ \Omega$.



$$R_{1,2} = \frac{(3\ \Omega)(6\ \Omega)}{3\Omega + 6\ \Omega} = 2\ \Omega; \quad R_{1,2,3} = 2\ \Omega + 1\ \Omega = 3\ \Omega$$

$$R_e = \frac{(3\ \Omega)(2\ \Omega)}{3\Omega + 2\ \Omega} = 1.20\ \Omega;$$

$$R_e = 1.20\ \Omega + 0.4\ \Omega = 1.60\ \Omega$$

$$I_T = \frac{24\text{ V}}{1.60\ \Omega}; \quad I_T = 15.0\text{ A}$$

$$V_4 = V_{1.2} = (1.2\ \Omega)(15\text{ A}) = 18\text{ V} \quad I_4 = \frac{18\text{ V}}{2\ \Omega}$$

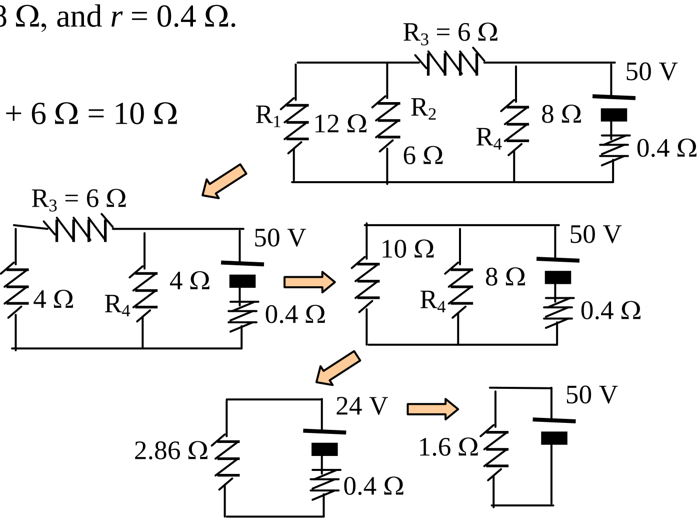
$I_4 = 9.0$ A; $I_3 = 15\text{ A} - 9\text{ A} = 6$ A; $V_3 = (6\text{ A})(1\ \Omega) = 6$ V; $V_1 = V_2 = 18\text{ V} - 6\text{ V}$;

$$V_1 = V_2 = 12\text{ V}; \quad I_2 = \frac{12\text{ V}}{3\ \Omega} = 4\text{ A}; \quad I_1 = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A};$$

$$\boxed{I_T = 15\text{ A},\ I_1 = 2\text{ A},\ I_2 = 4\ \Omega,\ I_3 = 6\text{ A},\ I_4 = 9\text{ A}.}$$

The solution is easier using Kirchhoff's laws, developed later in this chapter.

*28-24. Find the total current and the current through each resistor for Fig. 28-21 when $\mathcal{E}$ = 50 V, R_1 = 12 Ω, R_2 = 6 Ω, R_3 = 6 Ω, R_4 = 8 Ω, and r = 0.4 Ω.



$$R_{1,2} = \frac{(12\,\Omega)(6\,\Omega)}{12\,\Omega + 6\,\Omega} = 4\,\Omega; \quad R_{1,2,3} = 4\,\Omega + 6\,\Omega = 10\,\Omega$$

$$R = \frac{(10\,\Omega)(8\,\Omega)}{10\,\Omega + 8\,\Omega} = 4.44\,\Omega$$

R_e = 4.44 Ω + 0.4 Ω = 4.84 Ω

$$I_T = \frac{50\text{ V}}{4.84\,\Omega}; \qquad I_T = 10.3\text{ A}$$

$$V_4 = V_p = (4.44\,\Omega)(10.3\text{ A}) = 45.9\text{ V} \qquad I_4 = \frac{45.9\text{ V}}{8\,\Omega}$$

I_4 = 5.73 A; I_3 = 10.3 A – 5.73 A = 4.59 A; V_3 = (4.59 A)(6 Ω) = 27.5 V;

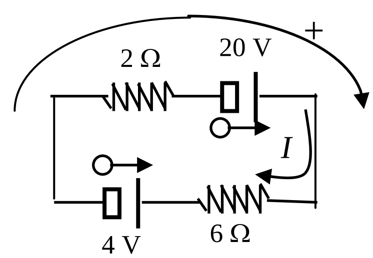
$$V_1 = V_2 = 45.9\text{ V} - 27.5\text{ V} = 18.4\text{ V}; \quad I_2 = \frac{18.4\text{ V}}{6\,\Omega} = 3.06\text{ A}; \quad I_1 = \frac{18.4\text{ V}}{12\,\Omega} = 1.53\text{ A};$$

I_T = 10.3 A, I_1 = 1.53 A, I_2 = 3.06 Ω, I_3 = 4.59 A, I_4 = 5.73 A.

Kirchhoff's Laws

28-25. Apply Kirchhoff's second rule to the current loop in Fig. 28-22. What is the net voltage around the loop? What is the net IR drop? What is the current in the loop?

Indicate output directions of emf's, assume direction of current, and trace in a clockwise direction for loop rule:



$\Sigma\mathcal{E} = \Sigma IR$; 20 V – 4 V = I(6 Ω) + I(2 Ω);

$$(8\,\Omega)I = 16\text{ V}; \qquad I = \frac{16\text{ V}}{8\,\Omega}; \quad \boxed{I = 2.00\text{ A}}$$

Net voltage drop = $\Sigma\mathcal{E}$ = $\boxed{16\text{ V};}$ ΣIR = (8 Ω)(2 A) = $\boxed{16\text{ V}}$

28-26. Answer the same questions for Problem 28-25 where the polarity of the 20-V battery is changed, that is, its output direction is now to the left? (*Refer to Fig. in Prob. 28-25*.)

$\Sigma\,\mathcal{E}$ = -20 V – 4 V = -24 V; ΣIR = I(2 Ω) + I(6 Ω = (8Ω)I

Σℰ = ΣIR; *-24 V* = (8 Ω)I; I = -4.00 A; ΣIR = (8 Ω)(-4 A) = -24 V

The minus sign means the current is counterclockwise (against the assume direction)

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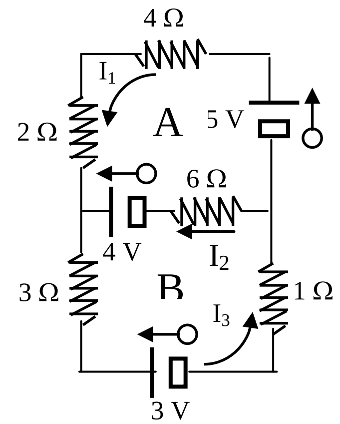
*28-27. Use Kirchhoff's laws to solve for the currents through the circuit shown as Fig. 28-23.



First law at point P: $I_1 + I_2 = I_3$ *Current rule*

Loop A (2nd law): Σℰ = Σ*IR* *Loop rule*

5 V – 4 V = (4 Ω)I_1 + (2 Ω)I_1 – (6 Ω)I_2

Simplifying we obtain: (1) $6I_1 - 6I_2 = 1$ A

Loop B: 4 V – 3 V = (6 Ω)I_2 + (3 Ω)I_3 + (1 Ω)I_3

Simplifying: (2) $6I_2 + 4I_3 = 1$ A, *but* $I_3 = I_1 + I_2$

Substituting we have: $6I_2 + 4(I_1 + I_2) = 1$ A *or* (3) $4I_1 + 10I_2 = 1$ A

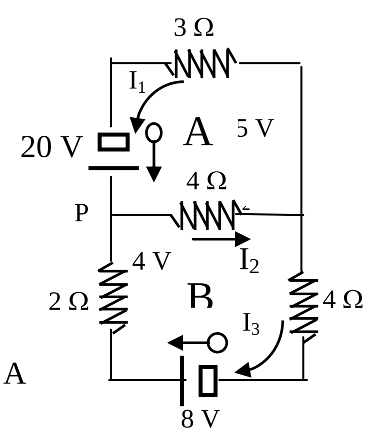
From which, I_1 = 0.25 A – 2.5 I_2 ; *Substituting into* (1): 6(0.25 A – 2.5I_2) – 6 I_2 = 1 A

1.5 A – 15I_2 – 6I_2 = 1 A; -21I_2 = -0.5 A; I_2 = 0.00238 *A;* I_2 = 23.8 mA

Putting this into (1), we have: 6I_1 – 6(0.0238 A) = 1 A, *and* I_1 = 190 mA

Now, $I_1 + I_2 = I_3$ *so that* I_3 *= 23.8 mA + 190 mA* *or* I_3 = 214 mA

*28-28. Use Kirchhoff's laws to solve for the currents in Fig. 28-24.



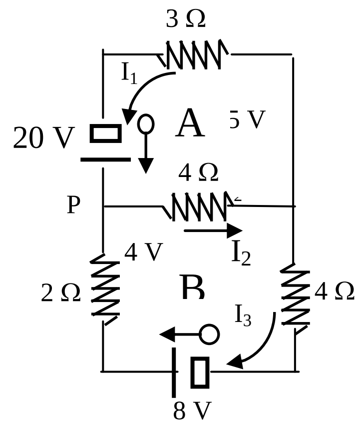
Current rule: $I_1 + I_3 = I_2$ *or* (1) $I_3 = I_2 - I_1$

Loop A: 20 V = (3 Ω)I_1 + (4 Ω)I_2 ; (2) $3I_1 + 4I_2 = 20$ A

Loop B: 8 V = (6 Ω)I_3 + (4 Ω)I_2; (3) $3I_3 + 2I_2 = 4$ A

Outside Loop: 20 V – 8 V = (3 Ω)I_1 – (6 Ω)I_3 or $I_1 - 2\,I_3 = 4$ A

*28-28. (Cont.) (3) $3I_3 + 2I_2 = 4$ A *and* $I_3 = I_2 - I_1$



$3(I_2 - I_1) + 2I_2 = 4$ A; $3I_1 = 5I_2 - 4$ A

(2) $3I_1 + 4I_2 = 20$ A; $(5I_2 - 4\text{ A}) + 4I_2 = 20$;

$I_2 = 2.67$ A; $3I_1 = 5(2.67\text{ A}) - 4$ A; $I_1 = 3.11$ A

$I_3 = I_2 - I_1 = 2.67\text{ A} - 3.11\text{ A} = -0.444$ A

*Note: I_3 goes in opposite direction to that ass*umed.

$I_1 = 3.11$ A, $I_2 = 2.67$ A, $I_3 = 0.444$ A

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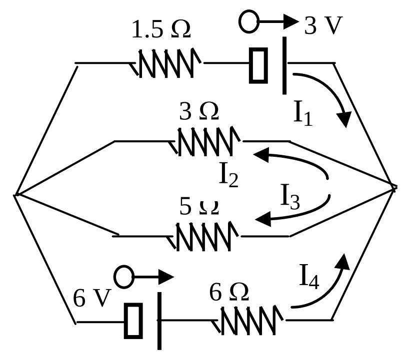
*28-29. Apply Kirchhoff's laws to the circuit of Fig. 28-25. Find the currents in each branch.



Current rule: (1) $I_1 + I_4 = I_2 + I_3$

Applying loop rule gives six possible equations:

(2) $1.5I_1 + 3I_2 = 3$ A; (3) $3I_2 - 5I_3 = 0$

(4) $5I_3 + 6I_4 = 6$A; (5) $1.5I_1 - 6I_4 = -3$A

(6) $6I_4 + 3I_2 = 6$ A (7) $1.5I_1 + 5I_3 = 3$A

Put $I_4 = I_2 + I_3 - I_1$; *into (4):* $5I_3 + 6(I_2 + I_3 - I_1) = 6$ A → $-6I_1 + 6I_2 + 11I_3 = 6$ A

Now, solving (2) for I_1 gives: $I_1 = 2\text{ A} - 2I_2$, *which can be used in the above equation.*

$-6(2\text{ A} - 2I_2) + 6I_2 + 11I_3 = 6$ A, *which gives:* $18I_2 + 11I_3 = 18$ A

But, from (3), we put $I_2 = \frac{5}{3} I_3$ *into above equation to find that:* $I_3 = 0.439$ A

From (2): $1.5I_1 + 3(0.439\text{ A}) = 3\text{ A}$; *and* $I_1 = 0.536$ A

From (3): $3I_2 - 5(0.439\text{ A}) = 0$; and $I_2 = 0.736$ A

From (4): $5(0.439\text{ A}) + 6I_4 = 6$ A; and $I_4 = 0.634$ A

Currents in each branch are: $I_1 = 536$ mA, $I_2 = 732$ mA, $I_3 = 439$ mA, $I_4 = 634$ mA

Note: Not all of the equations are independent. Elimination of two may yield another.

It is best to start with the current rule, and use it to eliminate one of the currents quickly.

The Wheatstone Bridge

28-30. A Wheatstone bridge is used to measure the resistance R_x of a coil of wire. The resistance box is adjusted for 6 Ω, and the contact key is positioned at the 45 cm mark when measured from point *A* of Fig. 28-13. Find R_x. (Note: $l_1 + l_2 = 100$ cm)

$$R_x = \frac{R_3 l_2}{l_1} = \frac{(6\ \Omega)(55\text{ cm})}{(45\text{ cm})}; \quad R_x = 7.33\ \Omega$$

28-31. Commercially available Wheatstone bridges are portable and have a self-contained galvanometer. The ratio R_2/R_1 can be set at any integral power of ten between 0.001 and 1000 by a single dual switch. When this ratio is set to 100 and the known resistance R is adjusted to 46.7 Ω, the galvanometer current is zero. What is the unknown resistance?

$$R_x = R_3 \frac{R_2}{R_1} = (46.7\ \Omega)(100); \quad \boxed{R_x = 4670\ \Omega}$$

28-32. In a commercial Wheatstone bridge, R_1 and R_2 have the resistances of 20 and 40 Ω, respectively. If the resistance R_x is 14 Ω, what must be the known resistance R_3 for zero galvanometer deflection?

$$R_x = R_3 \frac{R_2}{R_1} = (14\ \Omega)\frac{20\ \Omega}{40\ \Omega}; \quad \boxed{R_x = 7.00\ \Omega}$$

Challenge Problems:

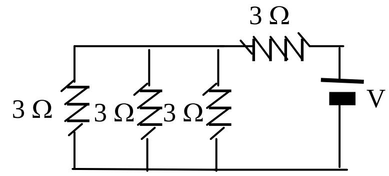
28-33. Resistances of 3, 6, and 9 Ω are first connected in series and then in parallel with an 36-V source of potential difference. Neglecting internal resistance, what is the current leaving the positive terminal of the battery?

$$R_e = \Sigma R_i = 3\ \Omega + 6\ \Omega + 9\ \Omega = 18\ \Omega; \quad I = \frac{36\text{ V}}{18\ \Omega}; \quad \boxed{I = 2.00\text{ A}}$$

28-33. (Cont.) $\frac{1}{R_e} = \sum \frac{1}{R_i} = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{9\,\Omega}$; $R_e = 1.64\ \Omega$; $I = \frac{36\text{ V}}{1.64\,\Omega}$; $\boxed{I = 22.0\text{ A}}$

28-34. Three 3-Ω resistors are connected in parallel. This combination is then placed in series with another 3-Ω resistor. What is the equivalent resistance?

$$\frac{1}{R'} = \frac{1}{3\,\Omega} + \frac{1}{3\,\Omega} + \frac{1}{3\,\Omega} = 1\,\Omega; \qquad R_e = R' + 3\,\Omega$$



$$R_e = 1\,\Omega + 3\,\Omega; \quad \boxed{R_e = 4\,\Omega}$$

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*28-35. Three resistors of 4, 8, and 12 Ω are connected in series with a battery. A switch allows the battery to be connected or disconnected from the circuit? When the switch is open, a voltmeter across the terminals of the battery reads 50 V. When the switch is closed, the voltmeter reads 48 V. What is the internal resistance in the battery?

$$R_L = 4\,\Omega + 8\,\Omega + 12\,\Omega = 24\,\Omega; \quad \mathcal{E} = 50\text{ V}; \quad V_T = 48\text{ V} = IR_L$$

$$I = \frac{48\text{ V}}{24\text{ V}} = 2.00\text{ A}; \quad \mathcal{E} - V_T = Ir; \quad 50\text{ V} - 48\text{ V} = Ir$$

$$r = \frac{50\text{ V - }48\text{ V}}{2.00\text{ A}}; \quad \boxed{r = 1.00\,\Omega}$$

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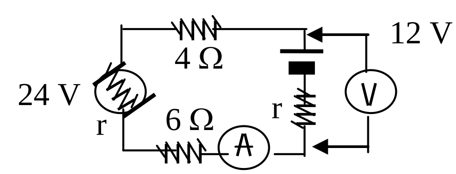
*28-36. The generator in Fig. 28-26 develops an emf of $\mathcal{E}_1 =$ 24 V and has an internal resistance of 0.2 Ω. The generator is used to charge a battery $\mathcal{E}_2 =$ 12 V whose internal resistance is 0.3 Ω. Assume that $R_1 = 4\ \Omega$ and $R_2 = 6\ \Omega$. What is the terminal voltage across the generator? What is the terminal voltage across the battery?

$$I = \frac{24\text{ V - }12\text{ V}}{6\,\Omega + 4\,\Omega + 0.2\,\Omega + 0.3\,\Omega} = 1.14\text{ A}$$

$$V_1 = \mathcal{E}_1 - Ir = 24\text{ V} - (1.14\text{ A})(0.2\,\Omega) = \boxed{23.8\text{ V};}$$

$$V_2 = 12\text{ V} + (1.14\text{ A})(0.3\,\Omega) = \boxed{12.3\text{ V}}$$

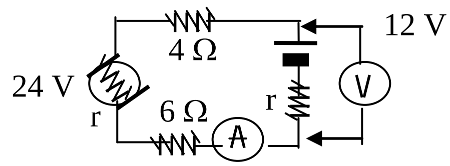


*28-37. What is the power consumed in charging the battery for Problem 28-36. Show that the power delivered by the generator is equal to the power losses due to resistance and the power consumed in charging the battery.

$P = \mathcal{E}I = (24\text{ V})(1.143\text{ A})$ $P_e = 27.43$ W

$P_R = I^2 R_e = (1.143\text{ A})^2 (10.5\ \Omega)$; $P_R = 13.69$ W

$P_V = (12\text{ V})(1.143\text{ A}) = 13.72$ W; $P_e = P_R + P_V$;



27.4 W = 13.7 W + 13.7 W

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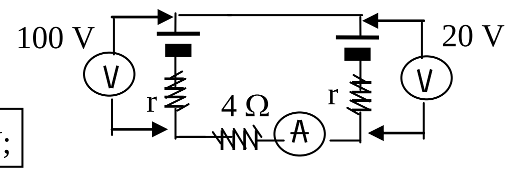
*28-38. Assume the following values for the parameters of the circuit illustrated in Fig. 28-8: $\mathcal{E}_1 = 100$ V, $\mathcal{E}_2 = 20$ V, $r_1 = 0.3\ \Omega$. $r_2 = 0.4\ \Omega$, and $R = 4\ \Omega$. What are the terminal voltages V_1 and V_2? What is the power lost through the 4-Ω resistor?

$$I = \frac{100\text{ V} - 20\text{ V}}{4\ \Omega + 0.3\ \Omega + 0.4\ \Omega} = 17.0\text{ A}$$



$V_1 = \mathcal{E}_1 - Ir = 100\text{ V} - (17.0\text{ A})(0.3\ \Omega) =$ **94.9 V;**

$V_2 = 20\text{ V} + (17.0\text{ A})(0.4\ \Omega) =$ **26.8 V;** $P = I^2 R = (17\text{ A})^2(4\ \Omega) =$ **1160 W**

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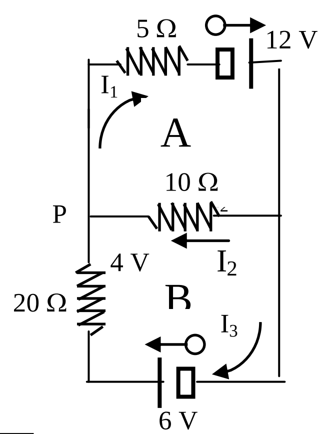
*28-39. Solve for the currents in each branch for Fig. 28-27.



Current rule: $I_1 = I_2 + I_3$; *Loops:* $\Sigma\mathcal{E} = \Sigma IR$*'s*

(1) $5I_1 + 10I_2 = 12$ A; (2) $-10I_2 + 20I_3 = 6$ A

(2) $-5I_2 + 10I_3 = 3$ A; (3) $5I_1 + 20I_3 = 18$ A

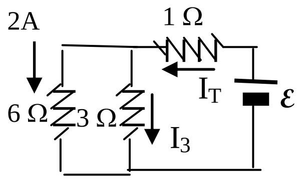
From (1): $5(I_2 + I_3) + 10 I_2 = 12\text{ A} \rightarrow 15I_2 + 5I_3 = 12$ A

Multiplying this equation by –2: $-30I_2 - 10I_3 = -24$ A

Now add this to (2): $-35I_2 + 0 = -21$ A and **$I_2 = 0.600$ A**

Now, from (1) and from (2): **$I_1 = 1.20$ A** and **$I_3 = 0.600$ A**

*28-40. If the current in the 6-Ω resistor of Fig. 28-28 is 2 A, what is the emf of the battery? Neglect internal resistance. What is the power loss through the 1-Ω resistor?



V_6 = (2 A)(6 Ω) = 12 V; $V_3 = 12\text{ V} = I_3(3\ \Omega)$

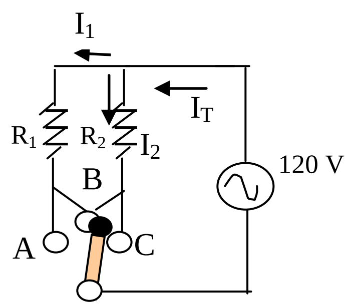
$$I_3 = \frac{12\text{ V}}{3\ \Omega} = 4\text{ A}; \quad I_T = 2\text{ A} + 4\text{ A} = 6\text{ A};$$

V_1 = (1 Ω)(6 A) = 6 V; $\mathcal{E}$ = 6 V + 12 V = **18 V**; $P = I_T^2 R = (6\text{ A})^2(1\ \Omega) =$ **36 W**

Critical Thinking Problems

*28-41. A three-way light bulb uses two resistors, a 50-W filament and a 100-W filament. A three-way switch allows each to be connected in series and provides a third possibility by connecting the two filaments in parallel? Draw a possible arrangement of switches than will accomplish these tasks. Assume that the household voltage is 120 V. What are the resistances of each filament? What is the power of the parallel combination?

Switch can be set at A, B, or C to give three possibilities:

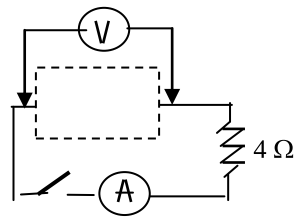


$$P = \frac{V^2}{R}; \quad R_1 = \frac{(120\text{ V})^2}{50\text{ W}} = \boxed{288\ \Omega;} \quad R_2 = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

For Parallel: $$R_e = \frac{(288\ \Omega)(144\ \Omega)}{288\ \Omega + 144\ \Omega} = 96\ \Omega$$

$$P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{96\ \Omega}; \quad \boxed{P = 150\text{ W}}$$

*28-42. The circuit illustrated in Fig. 28-7 consists of a 12-V battery, a 4-Ω resistor, and a switch. When new, the internal resistance of the battery is 0.4 Ω, and a voltmeter is placed across the terminals of the battery. What will be the reading of the voltmeter when the switch is open and when it is closed? After a long period of time, the experiment is repeated and it is noted that open circuit reading is unchanged, but the terminal voltage has reduced by 10 percent. How do you explain the lower terminal voltage? What is the internal resistance of the old battery?



When new: $I = \dfrac{\mathcal{E}}{R+r} = \dfrac{12\text{ V}}{4\ \Omega + 0.4\ \Omega}$; $\boxed{I = 2.73\text{ A}}$

$V_T = \mathcal{E} - Ir = 12\text{ V} - (2.73\text{ A})(0.4\ \Omega)$; $V_T = 10.9\text{ V}$

V_T reduced by 10% due to increase of $r_{int.}$ $V' = 10.9\text{ V} - 0.1(10.9\text{ V})$; $V' = 9.81\text{ V}$

$V' = IR_L = 9.81\text{ V}$; $I = \dfrac{9.81\text{ V}}{4\ \Omega} = 2.45\text{ A}$; $V' = \mathcal{E} - Ir$; $r = \dfrac{\mathcal{E} - V'}{I}$

$$r = \frac{\mathcal{E} - V'}{I} = \frac{12\text{ V} - 9.81\text{ V}}{2.45\text{ A}}; \quad \boxed{r = 0.893\ \Omega}$$

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*28-43. Given three resistors of 3, 9, and 18 Ω, list all the possible equivalent resistances that can be obtained through various connections?

All in parallel: $\dfrac{1}{R_e} = \dfrac{1}{3\ \Omega} + \dfrac{1}{9\ \Omega} + \dfrac{1}{18\ \Omega}$; $\boxed{R_e = 2\ \Omega}$

All in series: $R_e = R_1 + R_2 + R_3 = 3\ \Omega + 9\ \Omega + 18\ \Omega$; $\boxed{R_e = 30\ \Omega}$

Parallel (3,9) in series with (18): $R_e = \dfrac{(3\ \Omega)(9\ \Omega)}{3\ \Omega + 9\ \Omega} + 18\ \Omega$; $\boxed{R_e = 20.2\ \Omega}$

Parallel (3,18) in series with (9): $R_e = \dfrac{(3\ \Omega)(18\ \Omega)}{3\ \Omega + 18\ \Omega} + 9\ \Omega$; $\boxed{R_e = 11.6\ \Omega}$

Parallel (9,18) in series with (3): $R_e = \dfrac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} + 3\ \Omega$; $\boxed{R_e = 9.00\ \Omega}$

*28-43 (Cont.) *Series (3 + 9) in parallel with (18):* $R_e = \dfrac{(12\ \Omega)(18\ \Omega)}{12\ \Omega + 18\ \Omega}$; $\boxed{R_e = 20.2\ \Omega}$

Series (3 + 18) in parallel with (9): $R_e = \dfrac{(9\ \Omega)(21\ \Omega)}{9\ \Omega + 21\ \Omega}$; $\boxed{R_e = 6.30\ \Omega}$

Series (9 + 18) in parallel with (3): $R_e = \dfrac{(3\ \Omega)(27\ \Omega)}{3\ \Omega + 27\ \Omega}$; $\boxed{R_e = 2.70\ \Omega}$

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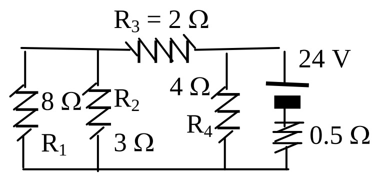
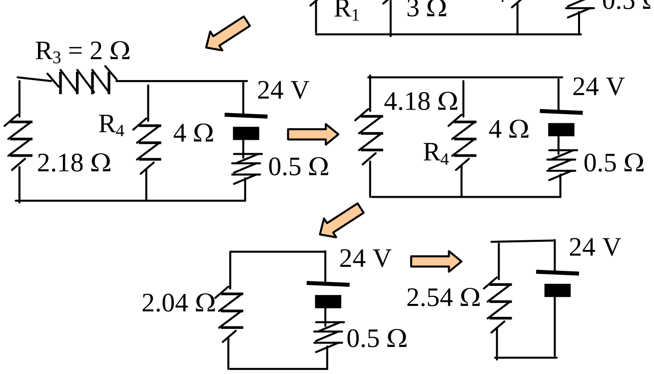
*28-44. Refer to Fig. 28-21, assume that $\mathcal{E} = 24$ V, $R_1 = 8\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 4\ \Omega$, and $r = 0.5\ \Omega$. What current is delivered to the circuit by the 24-V battery? What are the voltage and current for the 8-Ω resistor?



$R_{1,2} = \dfrac{(3\ \Omega)(8\ \Omega)}{3\ \Omega + 8\ \Omega} = 2.18\ \Omega$; $R_{1,2,3} = 2.18\ \Omega + 2\ \Omega = 4.18\ \Omega$



$R_e = \dfrac{(4.18\ \Omega)(4\ \Omega)}{4.18\ \Omega + 4\ \Omega} = 2.04\ \Omega$;

$R_e = 2.04\ \Omega + 0.5\ \Omega = 2.54\ \Omega$

$I_T = \dfrac{24\text{ V}}{2.54\ \Omega}$; $I_T = 9.43$ A

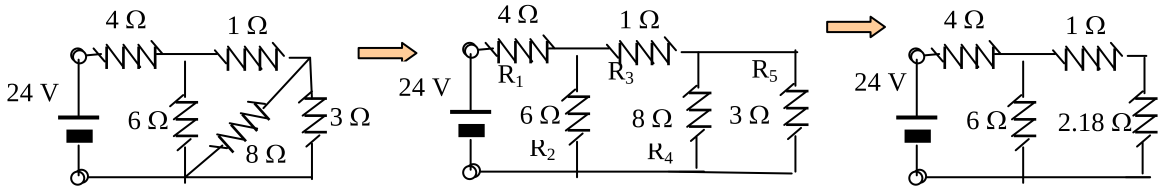
$V_4 = V_{1,2,3} = (9.43\text{ A})(2.04\ \Omega) = 19.3$ V

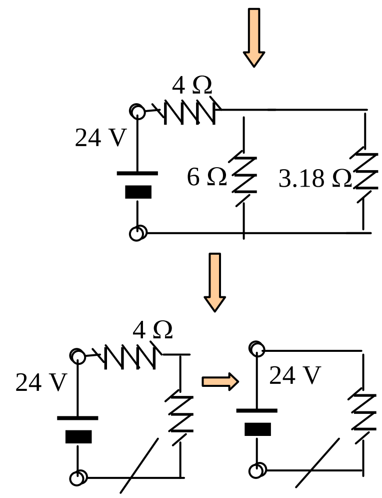
$I_4 = \dfrac{19.3\text{ V}}{4\ \Omega} = 4.82$ A; $I_3 = 9.43\text{ A} - 4.82\text{ A} = 4.61$ A; $V_3 = (4.61\text{ A})(2\ \Omega) = 9.23$ V

$V_1 = V_2 = 19.3\text{ V} - 9.23\text{ V}$; $V_1 = V_2 = 10.1$ V; $I_1 = \dfrac{10.1\text{ V}}{8\ \Omega} = 1.26$ A;

Finally, for the 8-Ω resistor: $\boxed{V_1 = 10.1\text{ V } \textit{and } I_1 = 1.26\text{ A}}$

*28-45. What is the effective resistance of the external circuit for Fig. 28-29 if internal resistance is neglected. What is the current through the 1-Ω resistor?





$$R_{4,5} = \frac{(3\ \Omega)(8\ \Omega)}{3\ \Omega + 8\ \Omega} = 2.18\ \Omega; \qquad R_{3,4,5} = 2.18\ \Omega + 1\ \Omega = 3.18\ \Omega$$

$$R_{1,2,3,4} = \frac{(3.18\ \Omega)(6\ \Omega)}{3.18\ \Omega + 6\ \Omega}; \qquad R_{1,2,3,4} = 2.08\ \Omega$$

$$R_e = 2.08\ \Omega + 4\ \Omega; \qquad \boxed{R_e = 6.08\ \Omega}$$

I_3 in the 1-Ω *resistor is same as I in $R_{3,4,5}$*

$$I_T = \frac{24\ \text{V}}{6.08\ \Omega} = 3.95\ \text{A}$$

$$V_{2,3,4,5} = (3.95\ \text{A})(2.08\ \Omega) = 8.21\ \text{V}; \qquad \textit{Also} \quad V_{3,4,5} = 8.21\ \text{V}$$

$$I_{3,4,5} = \frac{8.21\ \text{V}}{3.18\ \Omega} = 2.58\ \text{A}; \qquad \textit{Therefore} \quad I_3 = 2.58\ \text{A} \ \textit{in 1-}\Omega\ \textit{resistor}$$

$$\boxed{R_e = 6.08\ \Omega; \quad I_3 = 2.58\ \text{A}}$$