Chapter 27. Current and Resistance

Electric Current and Ohm's Law

27-1. How many electrons pass a point every second in a wire carrying a current of 20 A? How much time is needed to transport 40 C of charge past this point?

$$Q = It = (20\text{ C/s})(1\text{ s});\quad Q = 20\,\frac{\text{C}}{\text{s}}\left(\frac{1\,e}{1.6\text{ x }10^{-19}\text{C}}\right);\quad \boxed{\text{Q} = 1.25\text{ x }10^{20}\text{ electrons/s}}$$

$$I = \frac{Q}{t};\quad t = \frac{Q}{I} = \frac{40\text{ C}}{20\text{ A}} = 2.00\text{ s};\quad \boxed{t = 2.00\text{ s}}$$

27-2. If 600 C of charge pass a given point in 3 s, what is the electric current in amperes?

$$I = \frac{Q}{t} = \frac{600\text{ C}}{3\text{ s}};\quad \boxed{I = 20\text{ A}}$$

27-3. Find the current in amperes when 690 C of charge pass a given point in 2 min.

$$I = \frac{Q}{t} = \frac{690\text{ C}}{120\text{ s}};\quad \boxed{I = 5.75\text{ A}}$$

27-4. If a current of 24 A exists for 50 s, how many coulombs of charge have passed through the wire?

$$Q = It = (24\text{ A})(50\text{ s});\quad \boxed{Q = 1200\text{ C}}$$

27-5. What is the potential drop across a 4-Ω resistor with a current of 8 A passing through it?

$$V = IR = (8\text{ A})(4\ \Omega);\quad \boxed{V = 32.0\text{ V}}$$

27-6. Find the resistance of a rheostat if the drop in potential is 48 V and the current is 4 A.

$$R = \frac{V}{I} = \frac{48\text{ V}}{4\text{ A}};\quad \boxed{R = 12.0\ \Omega}$$

27-7. Determine the current through a 5-Ω resistor that has a 40-V drop in potential across it?

$$I = \frac{V}{R} = \frac{40\text{ V}}{5\ \Omega}; \qquad \boxed{I = 8.00\text{ A}}$$

27-8. A 2-A fuse is placed in a circuit with a battery having a terminal voltage of 12 V. What is the minimum resistance for a circuit containing this fuse?

$$R = \frac{V}{I} = \frac{12\text{ V}}{2\text{ A}}; \qquad \boxed{R = 6.00\ \Omega}$$

7-9. What emf is required to pass 60 mA through a resistance of 20 kΩ? If this same emf is applied to a resistance of 300 Ω, what will be the new current?

$$\mathcal{E} = IR = 60 \text{ x } 10^{-3}\text{ A})(20 \text{ x } 10^{3}\ \Omega); \qquad \mathcal{E} = 1200\text{ V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{1200\text{ V}}{300\ \Omega}; \qquad \boxed{I = 4.00\text{ A}}$$

Electric Power and Heat Loss

27-10. A soldering iron draws 0.75 A at 120 V. How much energy will it use in 15 min?

$$P = IV = (0.75\text{ A})(120\text{ V}); \quad P = 90.0\text{ W}; \quad t = 15\text{ min} = 900\text{ s}$$

$$P = \frac{Work}{t}; \qquad Energy = Pt = (90\text{ W})(900\text{ s}); \quad \boxed{E = 81{,}000\text{ J}}$$

27-11. An electric lamp has an 80-Ω filament connected to a 110-V direct-current line. What is the current through the filament? What is the power loss in watts?

$$\text{I} = \frac{V}{R} = \frac{110\text{ V}}{80\ \Omega}; \qquad \boxed{\text{I} = 1.38\text{ A}}$$

$$P = \frac{V^2}{R} = \frac{(110\text{ V})^2}{80\ \Omega}; \qquad \boxed{P = 151\text{ W}}$$

27-12. Assume that the cost of energy in a home is 8 cents per kilowatt-hour. A family goes on a 2-week vacation leaving a single 80-W light bulb burning. What is the cost?

$$E = Pt = (80\text{ W})(2\text{ wk})(7\text{ day/wk})(24\text{ h/day})(3600\text{ s/h}) = 26.9\text{ kW h}$$

$$E = (26.9\text{ kW h})(0.08\text{ c/kw h}) = \boxed{\$2.15} \quad \textit{(Rates vary)}$$

27-13. A 120-V, direct-current generator delivers 2.4 kW to an electric furnace. What current is supplied? What is the resistance?

$$I = \frac{P}{V} = \frac{2400\text{W}}{120\text{ V}}; \quad \boxed{I = 20\ A;} \quad R = \frac{V}{I} = \frac{120\text{ V}}{20\text{ A}}; \quad \boxed{R = 6.00\ \Omega}$$

27-14. A resistor develops heat at the rate of 250 W when the potential difference across its ends is 120 V. What is its resistance?

$$P = \frac{V^2}{R}; \quad R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{250\text{ W};} \quad \boxed{R = 57.6\ \Omega}$$

27-15. A 120-V motor draws a current of 4.0 A. How many joules of electrical energy is used in one hour? How many kilowatt-hours?

$$P = VI = (120\text{ V})(4.0\text{ A}) = 480\text{ W}$$

$$P = \frac{E}{t}; \quad E = Pt = (480\text{ W})(3600\text{ s}); \quad \boxed{E = 1.73\text{ MJ}}$$

$$E = 1.73 \text{ x } 10^6\text{ J}\left(\frac{1\text{ kW}\cdot\text{h}}{3.60 \text{ x } 10^6\text{ J}}\right) \quad \boxed{E = 0.480\text{ kW h}}$$

27-16. A household hair dryer is rated at 2000 W and is designed to operate on a 120-V outlet. What is the resistance of the device?

$$P = \frac{V^2}{R}; \quad R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{2000\text{ W}}; \quad \boxed{R = 7.20\ \Omega}$$

Resistivity

27-17. What length of copper wire 1/16 in. in diameter is required to construct a 20-Ω resistor at 20^0C? What length of nichrome wire is needed?

Copper: ρ = 1.78 x 10^{-8} Ω m ; *nichrome*: ρ = 100 x 10^{-8} Ω m

$$\tfrac{1}{16}\text{ ft} = 0.0625\text{ in.} = 62.5\text{ mil};\qquad A = (62.5\text{ mil})^2 = 3906\text{ cmil}$$

$$R = \frac{\rho l}{A};\quad l = \frac{RA}{\rho} = \frac{(20\ \Omega)(3906\text{ cmil})}{10.4\ \Omega\cdot\text{cmil/ft}};\quad \boxed{l = 7510\text{ ft}}$$

$$R = \frac{\rho l}{A};\quad l = \frac{RA}{\rho} = \frac{(20\ \Omega)(3906\text{ cmil})}{600\ \Omega\cdot\text{cmil/ft}};\quad \boxed{l = 130\text{ ft}}$$

27-18. A 3.0-m length of copper wire (ρ = 1.78 x 10^{-8} Ω m) at 20^0C has a cross section of 4 mm^2.What is the electrical resistance of this wire? [A = 4 mm^2 = 4 x 10^{-6} m^2]

$$R = \frac{\rho l}{A} = \frac{(1.72\text{ x }10^{-8}\Omega\cdot\text{m})(3.0\text{ m})}{4.00\text{ x }10^{-6}\text{m}^2};\quad \boxed{R = 12.9\text{ m}\Omega}$$

27-19. Find the resistance of 40 m of tungsten (ρ = 5.5 x 10^{-8} Ω m) wire having a diameter of 0.8 mm at 20^0C?

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.0008\text{ m})^2}{4};\quad A = 5.03\text{ x }10^{-7}\text{ m}^2$$

$$R = \frac{\rho l}{A} = \frac{(5.5\text{ x }10^{-8}\Omega\cdot\text{m})(40.0\text{ m})}{5.03\text{ x }10^{-7}\text{m}^2};\quad \boxed{R = 4.37\ \Omega}$$

27-20. A certain wire has a diameter of 3 mm and a length of 150 m. It has a resistance of 3.00 Ω at 20^0C. What is the resistivity? [A = $\pi D^2/4$ = 7.07 x 10^{-7} m^2.]

$$R = \frac{\rho l}{A};\quad \rho = \frac{RA}{l} = \frac{(3\ \Omega)(7.07\text{ x }10^{-7}\text{m}^2)}{150\text{ m}};\quad \boxed{\rho = 1.41\text{ x }10^{-8}\ \Omega\text{ m}}$$

27-21. What is the resistance of 200 ft of iron ($\rho = 9.5 \times 10^{-8}\ \Omega$ m) wire with a diameter of 0.002 in. at 20^0C? ($\rho = 9.5 \times 10^{-8}\ \Omega$ m). [200 ft = 61.0 m; 0.002 in. = 5.08×10^{-5} m]

$$A = \frac{\pi D^2}{4} = \frac{\pi(5.08 \text{ x } 10^{-5} \text{ m})^2}{4}; \qquad A = 2.03 \text{ x } 10^{-9} \text{ m}^2$$

$$R = \frac{\rho l}{A} = \frac{(9.5 \text{ x } 10^{-8}\Omega\cdot\text{m})(61.0 \text{ m})}{5.08 \text{ x } 10^{-5}\text{m}^2}; \quad \boxed{R = 2860\ \Omega}$$

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*27-22. A nichrome wire ($\rho = 100 \times 10^{-8}\ \Omega$ m) has a length of 40 m at 20^0C What is the diameter if the total resistance is 5 Ω?

$$R = \frac{\rho l}{A}; \quad A = \frac{\rho l}{R} = \frac{(100 \text{ x } 10^{-8}\Omega\cdot\text{m})(40 \text{ m})}{5.00\ \Omega}; \quad A = 8 \text{ x } 10^{-6} \text{ m}^2$$

$$A = \frac{\pi D^2}{4}; \quad D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{2(8 \text{ x } 10^{-6}\text{m}^2)}{\pi}}; \quad \boxed{D = 2.26 \text{ mm}}$$

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*27-23. A 115-V source of emf is attached to a heating element which is a coil of nichrome wire ($\rho = 100 \times 10^{-8}\ \Omega$ m) of cross section 1.20 mm² What must be the length of the wire if the resistive power loss is to be 800 W? [A = 1.20 mm² = 1.20×10^{-6} m²]

$$P = \frac{V^2}{R}; \quad R = \frac{V^2}{P} = \frac{(115 \text{ V})^2}{800 \text{ W}} = 16.5\ \Omega; \qquad R = 16.5\ \Omega$$

$$R = \frac{\rho l}{A}; \quad l = \frac{RA}{\rho} = \frac{(16.5\ \Omega)(1.20 \text{ x } 10^{-6}\text{m}^2)}{100 \text{ x } 10^{-8}\ \Omega\cdot\text{m}}; \quad \boxed{l = 19.8 \text{ m}}$$

Temperature Coefficient of Resistance

27-24. The resistance of a length of wire ($\alpha = 0.0065/C^0$) is 4.00 Ω at 20^0C. What is the resistance at 80^0C? [$\Delta t = 80^0\text{C} - 20^0\text{C} = 60\ \text{C}^0$]

$$\Delta R = \alpha R_o \Delta t = (0.0065/\text{C}^0)(4\ \Omega)(60\ \text{C}^0) = 1.56\ \Omega; \quad \text{R} = 4.00\ \Omega + 1.56\ \Omega = \boxed{5.56\ \Omega}$$

27-25. If the resistance of a conductor is 100 Ω at 20^0C, and 116 Ω at 60^0C, what is its temperature coefficient of resistivity? [$\Delta t = 60^0\text{C} - 20^0\text{C} = 40\ \text{C}^0$]

$$\alpha = \frac{\Delta R}{R_0 \Delta t} = \frac{116\ \Omega - 100\ \Omega}{(100\ \Omega)(40\ \text{C}^0)}; \quad \boxed{\alpha = 0.00400\ /\text{C}^0}$$

27-26. A length of copper (α = $0.0043/\text{C}^0$) wire has a resistance of 8 Ω at 20^0C. What is the resistance at 90^0C? At - 30^0C?

$$\Delta R = (0.0043/\text{C}^0)(8\ \Omega)(70\ \text{C}^0) = 2.41\ \Omega; \quad \text{R} = 8.00\ \Omega + 2.41\ \Omega = \boxed{10.41\ \Omega}$$

$$\Delta R = (0.0043/\text{C}^0)(8\ \Omega)(-30^0\text{C} - 20^0\text{C}) = -1.72\ \Omega; \quad \text{R} = 8.00\ \Omega - 1.72\ \Omega = \boxed{6.28\ \Omega}$$

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*27-27. The copper windings (α = $0.0043/\text{C}^0$) of a motor experience a 20 percent increase in resistance over their value at 20^0C. What is the operating temperature?

$$\frac{\Delta R}{R} = 0.2; \quad \Delta t = \frac{\Delta R}{R_0 \alpha} = \frac{0.2}{0.0043/C^0} = 46.5\ \text{C}^0; \ \ \text{t} = 20^0\text{C} + 46.5\,\text{C}^0 = \boxed{66.5\ ^0\text{C}}$$

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*27-28. The resistivity of copper at 20^0C is 1.78×10^{-8} Ω m. What change in temperature will produce a 25 percent increase in resistivity?

Challenge Problems

27-29. A water turbine delivers 2000 kW to an electric generator which is 80 percent efficient and has an output terminal voltage of 1200 V. What current is delivered and what is the electrical resistance? [$P_{out} = (0.80)(2000\ \text{kW}) = 1600\ \text{kW}$]

$$P = VI; \quad I = \frac{P}{V} = \frac{1600 \times 10^3\ \text{W}}{1200\ \text{V}}; \quad \boxed{I = 1330\ \text{A}}$$

$$R = \frac{V}{I} = \frac{1200\ \text{V}}{1300\ \text{A}}; \quad \boxed{\text{R} = 0.900\ \Omega}$$

27-30. A 110-V radiant heater draws a current of 6.0 A. How much heat energy in joules is delivered in one hour?

$$P = \frac{E}{t} = VI; \quad E = VIt = (110\text{ V})(6\text{ A})(3600\text{ s}); \quad \boxed{E = 2.38\text{ MJ}}$$

27-31. A power line has a total resistance of 4 kΩ. What is the power loss through the wire if the current is reduced to 6.0 mA?

$$P = I^2R = (0.006\text{ A})^2(4000\ \Omega); \quad \boxed{P = 144\text{ mW}}$$

27-32. A certain wire has a resistivity of 2 x 10^{-8} Ω m at 20^0C. If its length is 200 m and its cross section is 4 mm^2, what will be its electrical resistance at 100^0C. Assume that α = 0.005/C^0 for this material. [Δt = 100^0C – 20^0C = 80 C^0]

$$R_0 = \frac{\rho l}{A} = \frac{(2\text{ x }10^{-8}\Omega\cdot\text{m})(200\text{ m})}{4\text{ x }10^{-6}\text{m}^2}; \quad R_0 = 1.00\ \Omega \quad at\ 20^0\text{C}$$

$$R = R_0 + \alpha R_0 \Delta t = 1.00\ \Omega + (0.005/\text{C}^0)(1\ \Omega)(80\text{ C}^0); \quad \boxed{R = 1.40\ \Omega}$$

27-33. Determine the resistivity of a wire made of an unknown alloy if its diameter is 0.007 in. and 100 ft of the wire is found to have a resistance of 4.0 Ω. [D = 0.007 in. = 7 mil]

$$A = (7\text{ mil})^2 = 49\text{ cmil}; \quad R = \frac{\rho l}{A}; \quad \rho = \frac{RA}{l}$$

$$\Delta = \frac{RA}{l} = \frac{(4\ \Omega)(49\text{ cmil})}{100\text{ ft}}; \quad \boxed{\rho = 1.96\ \Omega\text{ cmil/ft}}$$

27-34. The resistivity of a certain wire is 1.72 x 10^{-8} Ω m at 20^0C. A 6-V battery is connected to a 20-m coil of this wire having a diameter of 0.8 mm. What is the current in the wire?

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.0008\text{ m})^2}{4} = 5.03\text{ x }10^{-7}\text{m}^2; \quad R = \frac{\rho l}{A}$$

27-34. (Cont.) $R = \frac{\rho l}{A} = \frac{(1.72 \text{ x } 10^{-8}\Omega\cdot\text{m})(20 \text{ m})}{5.03 \text{ x } 10^{-7}\text{m}^2}$; $R = 0.684\ \Omega$

$$I = \frac{V}{R} = \frac{6.00 \text{ V}}{0.684\ \Omega}; \quad \boxed{I = 8.77 \text{ A}}$$

27-35. A certain resistor is used as a thermometer. Its resistance at 20^0C is 26.00 Ω, and its resistance at 40^0C is 26.20 Ω. What is the temperature coefficient of resistance for this material?

$$\alpha = \frac{\Delta R}{R_0 \Delta t} = \frac{(26.20\ \Omega - 26.00\ \Omega)}{(26.00\ \Omega)(40^0\text{C} - 20^0\text{C})}; \quad \boxed{\alpha = 3.85 \text{ x } 10^{-4}/\text{C}^0}$$

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*27-36. What length of copper wire at 20^0C has the same resistance as 200 m of iron wire at 20^0C? Assume the same cross section for each wire. [*Product RA doesn't change*.]

$$R = \frac{\rho l}{A}; \quad RA = \rho l; \quad \rho_1 R_1 = \rho_2 l_2; \quad l_2 = \frac{\rho_1 l_1}{\rho_2}$$

$$l_2 = \frac{\rho_1 l_1}{\rho_2} = \frac{(9.5x10^{-8}\Omega\cdot\text{m})(200 \text{ m})}{1.72 \text{ x } 10^{-8}\Omega\cdot\text{m}}; \quad \boxed{l_2 = 1100 \text{ m}}$$

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*27-37. The power loss in a certain wire at 20^0C is 400 W. If α = 0.0036/C^0, by what percentage will the power loss increase when the operating temperature is 68^0C?

$$\Delta R = \alpha R_0 \Delta t; \quad \frac{\Delta R}{R} = (0.0036/\text{C}^0)(68^0\text{C} - 20^0\text{C}) = 0.173$$

Since $P = I^2R$, the power loss increases by same percentage: $\boxed{17.3\ \%}$

Critical Thinking Problems

27-38. A 150-Ω resistor at 20^0C is rated at 2.0 W maximum power. What is the maximum voltage that can be applied across the resistor with exceeding the maximum allowable power? What is the current at this voltage?

$$P = \frac{V^2}{R}; \quad V = \sqrt{PR} = \sqrt{(2.00\text{ W})(150\ \Omega)}; \quad \boxed{V = 17.3\text{ V}}$$

27-39. The current in a home is alternating current, but the same formulas apply. Suppose a fan motor operating a home cooling system is rated at 10 A for a 120-V line. How much energy is required to operate the fan for a 24-h period? At a cost of 9 cents per kilowatt-hour, what is the cost of operating this fan continuously for 30 days?

$$P = VI = (110\text{ V})((10\text{ A}) = 1100\text{ W}; \quad E = Pt = (1100\text{ W})(24\text{ h}) = 26.4\text{ kW h}$$

$$E = (26.4\text{ kW h})(3600\text{ s/h})(1000\text{ W/kW}); \quad \boxed{E = 95.0\text{ MJ}}$$

$$Cost = 26.4\frac{\text{kW}\cdot\text{h}}{\text{day}}\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)(30\text{ days}); \quad \boxed{Cost = \$53.36}$$

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*27-40. The power consumed in an electrical wire (α= 0.004/C^0) is 40 W at 20^0C. If all other factors are held constant, what is the power consumption when (a) the length is doubled, (b) the diameter is doubled, (c) the resistivity is doubled, and (d) the absolute temperature is doubled? (*Power loss is proportional to resistance R*)

$$R = \frac{\rho l}{A}; \quad P \propto l; \quad P \propto \frac{1}{A}; \quad P \propto \Delta R \propto \Delta T$$

(a) *Double length and double power loss;* $Loss = 2(40\text{ W}) = \boxed{80\text{ W}}$

(b) doubling diameter gives $4A_0$ and one-fourth power loss: $Loss = ¼(40\text{ W}) = \boxed{10\text{ W}}$

(c) *Doubling resistivity doubles resistance, and also doubles power loss:* $\boxed{\text{Loss} = 80\text{ W}}$

*27-40. (Cont.) *(d)* $T = (20^0 + 273^0) = 293$ K; $\Delta T = 2T - T = T$; $\Delta T = 293$ K $= 293$ C^0

If absolute temperature doubles, the new resistance is given by:

$$R = R_0(1 + \alpha\Delta T); \quad \frac{R}{R_0} = 1 + (0.004/\text{C}^0)(293\ \text{C}^0) = 2.172;$$

$$\frac{P}{P_0} = \frac{R}{R_0} = 2.172; \quad \text{Loss} = 2.172(40\ \text{W}); \quad \boxed{Loss = 86.9\ \text{W}}$$

This of course presumes that resistivity remains linear, which is not likely.

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*27-41. What must be the diameter of an aluminum wire if it is to have the same resistance as an equal length of copper wire of diameter 2.0 mm? What length of nichrome wire is needed to have the same resistance as 2 m of iron wire of the same cross section?

$$R = \frac{\rho l}{A}; \quad \frac{R}{l} = \frac{\rho}{A} = const.; \quad \frac{\rho_c}{A_c} = \frac{\rho_a}{A_a}; \quad A = \frac{\pi D^2}{4}; \quad \frac{\rho_c}{(D_c)^2} = \frac{\rho_a}{(D_c)^2}$$

$$D_a^2 = \frac{\rho_a D_c^2}{\rho_c}; \quad D_a = D_c\sqrt{\frac{\rho_c}{\rho_a}} = (2\ \text{mm})\sqrt{\frac{1.72\ \text{x}\ 10^{-8}\Omega\cdot\text{m}}{2.80\ \text{x}\ 10^{-8}\Omega\cdot\text{m}}}; \quad \boxed{D_a = 1.57\ \text{mm}}$$

$$R = \frac{\rho l}{A}; \quad RA = \rho l = const.; \quad \rho_n l_n = \rho_i l_i; \quad l_n = \frac{\rho_i l_i}{\rho_n}$$

$$l_n = \frac{\rho_i l_i}{\rho_n} = \frac{(1.72\ \text{x}\ 10^{-8}\Omega\cdot\text{m})}{(100\ \text{x}\ 10^{-8}\Omega\cdot\text{m})}; \quad \boxed{l_n = 1.72\ \text{cm}}$$

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*27-42. An iron wire (α = 0.0065/C^0) has a resistance of 6.00 Ω at 20^{0}C and a copper wire (α = 0.0043/C^0) has a resistance of 5.40 Ω at 20^{0}C. At what temperature will the two wires have the same resistance? [Conditions: $\alpha_i R_{oi}\Delta t_i - \alpha_c R_{0c}\Delta t_c$ = 6 Ω –5.4 Ω = -0.60.Ω.]

$$\Delta t = \frac{-0.600\ \Omega}{\alpha_i R_{oi} - \alpha_c R_{oc}} = \frac{-0.600\ \Omega}{(0.0065/\text{C}^0)(6.0\ \Omega) - (0.0043/\text{C}^0)(5.4\ \Omega)}; \quad \Delta t = -38.0\ \text{C}^0$$

$$\Delta t = t_f - 20^0 = -38.0\ \text{C}^0; \quad \boxed{t_f = -18.0^0\text{C}}$$