Chapter 26. Capacitance

Capacitance

26-1. What is the maximum charge that can be placed on a metal sphere 30 mm in diameter and surrounded by air?

$$E = \frac{kQ}{r^2} = 3 \text{ x } 10^6 \text{N/C}; \quad Q = \frac{Er^2}{k} = \frac{(3 \text{ x } 10^6 \text{N/C})(0.015 \text{ m})^2}{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)}; \quad \boxed{Q = 75.0 \text{ nC}}$$

26-2. How much charge can be placed on a metal sphere of radius 40 mm if it is immersed in transformer oil whose dielectric strength is 16 MV/m?

$$E = \frac{kQ}{r^2} = 16 \text{ x } 10^6 \text{N/C}; \quad Q = \frac{Er^2}{k} = \frac{(16 \text{ x } 10^6 \text{N/C})(0.040 \text{ m})^2}{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)}; \quad \boxed{Q = 2.48 \ \mu\text{C}}$$

26-3. What would be the radius of a metal sphere in air such that it could theoretically hold a charge of one coulomb?

$$E = \frac{kQ}{r^2}; \quad r = \sqrt{\frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(1 \text{ C})}{3 \text{ x } 10^6 \text{ N/C}}}; \quad \boxed{r = 54.8 \text{ m}}$$

26-4. A 28-μF parallel-plate capacitor is connected to a 120-V source of potential difference. How much charge will be stored on this capacitor?

$$Q = CV = (28 \ \mu\text{F})(120 \text{ V}); \quad \boxed{Q = 3.36 \text{ mC}}$$

26-5. A potential difference of 110 V is applied across the plates of a parallel-plate capacitor. If the total charge on each plate is 1200 μC, what is the capacitance?

$$C = \frac{Q}{V} = \frac{1200 \ \mu\text{C}}{110 \text{ V}}; \quad \boxed{C = 10.9 \ \mu\text{F}}$$

26-6. Find the capacitance of a parallel-plate capacitor is 1600 μC of charge is on each plate when the potential difference is 80 V.

$$C = \frac{Q}{V} = \frac{1600\ \mu\text{C}}{80\ \text{V}}; \qquad \boxed{C = 20.0\ \mu\text{F}}$$

26-7. What potential difference is required to store a charge of 800 μC on a 40-μF capacitor?

$$V = \frac{Q}{C} = \frac{800\ \mu\text{C}}{40\ \mu\text{F}}; \qquad \boxed{V = 20.0\ \text{V}}$$

26-8. Write an equation for the potential at the surface of a sphere of radius r in terms of the permittivity of the surrounding medium. Show that the capacitance of such a sphere is given by $C = 4\pi\varepsilon r$.

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}; \quad Q = CV; \qquad \cancel{V} = \frac{C\cancel{V}}{4\pi\varepsilon_0 r}; \qquad \boxed{C = 4\pi\varepsilon_0 r}$$

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*26-9. A spherical capacitor has a radius of 50 mm and is surrounded by a medium whose permittivity is $3 \times 10^{-11}\ C^2/N\ m^2$. How much charge can be transferred to this sphere by a potential difference of 400 V?

We must replace ε_0 with ε for permittivity of surrounding medium, then

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon r}; \quad Q = CV; \qquad \cancel{V} = \frac{C\cancel{V}}{4\pi\varepsilon r}; \qquad \boxed{C = 4\pi\varepsilon r}$$

$$C = 4\pi\varepsilon r; \qquad Q = \frac{C}{V} = \frac{4\pi\varepsilon r}{V}$$

$$Q = \frac{4\pi(3 \times 10^{-11}\text{C}^2/\text{N}\cdot\text{m}^2)(0.05\ \text{m})}{400\ \text{V}}; \qquad \boxed{Q = 4.71 \times 10^{-14}\ \text{C}}$$

Calculating Capacitance

26-10. A 5-μF capacitor has a plate separation of 0.3 mm of air. What will be the charge on each plate for a potential difference of 400 V? What is the area of each plate?

$$Q = CV = (5\ \mu\text{F})(400\ \text{V}); \qquad \boxed{Q = 2000\ \mu\text{C}}$$

26-11. The plates of a certain capacitor are 3 mm apart and have an area of 0.04 m^2. What is the capacitance if air is the dielectric?

$$C = \varepsilon_0 \frac{E}{A} = \frac{(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.04\ \text{m}^2)}{0.003\ \text{m}}; \qquad \boxed{C = 118\ \text{pF}}$$

26-12. A capacitor has plates of area 0.034 m^2 and a separation of 2 mm in air. The potential difference between the plates is 200 V. What is the capacitance, and what is the electric field intensity between the plates? How much charge is on each plate?

$$C = \varepsilon_0 \frac{E}{A} = \frac{(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.034\ \text{m}^2)}{0.002\ \text{m}}; \qquad \boxed{C = 150\ \text{pF}}$$

$$E = \frac{V}{d} = \frac{400\ \text{V}}{0.002\ \text{m}}; \qquad \boxed{E = 2.00 \text{ x } 10^5\ \text{N/C}}$$

$$Q = CV = (150\ \text{pF})(400\ \text{V}); \qquad \boxed{Q = 30.1\ \mu\text{C}}$$

26-13. A capacitor of plate area 0.06 m^2 and plate separation 4 mm has a potential difference of 300 V when air is the dielectric. What is the capacitance for dielectrics of air (K = 1) and mica (K = 5)?

$$C = K\varepsilon_0 \frac{E}{A} = \frac{(1)(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.06\ \text{m}^2)}{0.004\ \text{m}}; \qquad \boxed{C = 133\ \text{pF}}$$

$$C = K\varepsilon_0 \frac{E}{A} = \frac{(5)(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.06\ \text{m}^2)}{0.004\ \text{m}}; \qquad \boxed{C = 664\ \text{pF}}$$

26-14. What is the electric field intensity for mica and for air in Problem 26-13?

$$E_0 = \frac{V_0}{d} \quad E_0 = \frac{V_0}{d} = \frac{300\text{ V}}{0.004\text{ m}}; \qquad \boxed{E_0 = 7.50 \times 10^4\text{ V/m}}$$

$$K = \frac{E_0}{E}; \qquad E = \frac{E_0}{K} = \frac{7.50 \times 10^4}{5}; \qquad \boxed{E = 1.50 \times 10^4\text{ V/m}}$$

26-15. Find the capacitance of a parallel-plate capacitor if the area of each plate is 0.08 m^2, the separation of the plates is 4 mm, and the dielectric is (a) air, (b) paraffined paper (K = 2)?

$$C = K\varepsilon_0 \frac{E}{A} = \frac{(1)(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.08\text{ m}^2)}{0.004\text{ m}}; \qquad \boxed{C = 177\text{ pF}}$$

$$C = K\varepsilon_0 \frac{E}{A} = \frac{(2)(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.08\text{ m}^2)}{0.004\text{ m}}; \qquad \boxed{C = 354\text{ pF}}$$

26-16. Two parallel plates of a capacitor are 4.0 mm apart and the plate area is 0.03 m^2. Glass (K = 7.5) is the dielectric, and the plate voltage is 800 V. What is the charge on each plate, and what is the electric field intensity between the plates?

$$C = K\varepsilon_0 \frac{E}{A} = \frac{(7.5)(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.03\text{ m}^2)}{0.004\text{ m}}; \qquad \boxed{C = 498\text{ pF}}$$

$$Q = CV = (498 \times 10^{-12}\text{ F})(800\text{ V}); \qquad \boxed{Q = 398\text{ nC}}$$

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*26-17. A parallel-plate capacitor with a capacitance of 2.0 nF is to be constructed with mica (K = 5) as the dielectric, and it must be able to withstand a maximum potential difference of 3000 V. The dielectric strength of mica is 200 MV/m. What is the minimum area the plates of the capacitor can have?

$$Q = CV = (2 \times 10^{-9}\text{F})(3000\text{ V}); \quad Q = 6\ \mu\text{C}; \quad d = \frac{V}{E} = \frac{3000\text{ V}}{200 \times 10^6\text{V/m}}$$

*16-17. (Cont.) d = 1.50 x 10^{-5} m; C = 2 nF; K = 5

$$C = K\varepsilon_0 \frac{A}{d}; \quad A = \frac{Cd}{K\varepsilon_0} = \frac{(2 \text{ x } 10^{-9}\text{F})(1.5 \text{ x } 10^{-5}\text{m})}{5(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)}; \quad \boxed{A = 6.78 \text{ x } 10^{-4} \text{ m}^2}$$

Capacitors in Series and in Parallel

26-18. Find the equivalent capacitance of a 6-μF capacitor and a 12-μF capacitor connected (a) in series, (b) in parallel.

Capacitors in series: $C_e = \frac{C_1C_2}{C_1 + C_2} = \frac{(6\ \mu\text{F})(12\ \mu\text{F})}{6\ \mu\text{F} + 12\ \mu\text{F}}$; $\boxed{C_e = 4.00\ \mu\text{F}}$

Capacitors in parallel: $C_e = C_1 + C_2$ = 6 μF + 12 μF; $\boxed{C_e = 18\ \mu\text{F}}$

26-19. Find the effective capacitance of a 6-μF capacitor and a 15-μF capacitor connected (a) in series, (b) in parallel?

Capacitors in series: $C_e = \frac{C_1C_2}{C_1 + C_2} = \frac{(6\ \mu\text{F})(15\ \mu\text{F})}{6\ \mu\text{F} + 15\ \mu\text{F}}$; $\boxed{C_e = 4.29\ \mu\text{F}}$

Capacitors in parallel: $C_e = C_1 + C_2$ = 6 μF + 12 μF; $\boxed{C_e = 21.0\ \mu\text{F}}$

26-20. What is the equivalent capacitance for capacitors of 4, 7, and 12 μF connected (a) in series, (b) in parallel?

Series: $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4\ \mu\text{F}} + \frac{1}{7\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}}$; $\boxed{C_e = 2.10\ \mu\text{F}}$

Parallel: $C_e = \Sigma C_i$ = 4 μF + 7 μC + 12 μC; $\boxed{C_e = 23.0\ \mu\text{C}}$

26-21. Find the equivalent capacitance for capacitors of 2, 6, and 8 μF connected (a) in series, (b) in parallel?

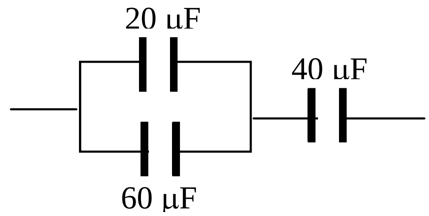
Series: $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} + \frac{1}{8\ \mu\text{F}}$; **$C_e$ = 1.26 μF**

Parallel: $C_e = \Sigma C_i$ = 2 μF + 6 μC + 8 μC; **C_e = 16.0 μC**

26-22. A 20-μF and a 60-μF capacitor are connected in parallel. Then the pair are connected in series with a 40-μF capacitor. What is the equivalent capacitance?

C' = 20 μF + 60 μF = 80 μF;

$$C_e = \frac{C'C_{40}}{C'+C_{40}} = \frac{(80\mu\text{F})(40\mu\text{F})}{(80\mu\text{F} + 40\mu\text{F})};$$ **C_e = 26.7 μF**



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*26-23. If a potential difference of 80 V is placed across the group of capacitors in Problem 26-22, what is the charge on the 40-μF capacitor? What is the charge on the 20-μF capacitor? (*First find total charge, then find charge and voltage on each capacitor.)*

$Q_T = C_eV = (26.7\ \mu\text{F})(80\ \text{V})$; $Q_T = 2133\ \mu\text{C}$; **Q_{40} = 2133 μC**

Note: 2133 μC are on each of the combination C' and the 40-μF capacitor. To find the charge across the 20 μF, we need to know the voltage across C':

$V_{C'} = \frac{2133\ \mu\text{C}}{80\ \mu\text{F}} =$ 26.7 V; *This is V across each of* 20 *and* 60-μF *capacitors*

Thus, charge on 20-μF capacitor is: Q_{20} = (20 μF)(26.7 V); **Q_{20} = 533 μC**

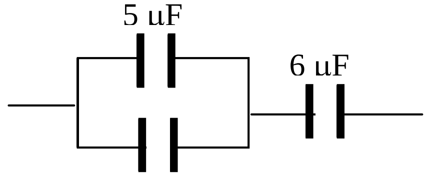
Note that V_{40} = 2133 μC/26.7 μC *or* 53.3 V. *Also* 53.3 V + 26.7 V = 80 V !

Also, the charge on the 60-μC capacitor is 1600 μC and 1600 μC + 533 μC = Q_T

*26-24. Find the equivalent capacitance of a circuit in which a 6-μF capacitor is connected in series with two parallel capacitors whose capacitances are 5 and 4 μF.

$C' = 5\ \mu F + 4\ \mu F = 9\ \mu F$;

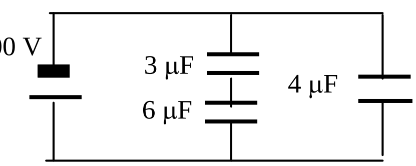
$$C_e = \frac{C'C_{40}}{C'+C_{40}} = \frac{(9\mu F)(6\mu F)}{9\mu F + 6\mu F};$$ **$C_e = 3.60\ \mu F$**



*26-25. What is the equivalent capacitance for the circuit drawn in Fig. 26-15?

$$C' = \frac{C_6C_3}{C_6+C_3} = \frac{(6\mu F)(3\mu F)}{6\mu F + 3\mu F};$$ $C' = 2.40\ \mu F$

$C_e = 2\ \mu F + 4\ \mu F$; **$C_e = 6.00\ \mu F$**



*26-26. What is the charge on the 4-μF capacitor in Fig. 26-15? What is the voltage across the 6-μF capacitor?

Total charge $Q_T = C_eV = (6.00\ \mu F)(200\ V)$; $Q_T = 1200\ \mu C$

$Q_4 = C_4V_4 = (4\ \mu C)(200\ V)$; $Q_4 = 800\ \mu C$

The rest of the charge is across EACH of other capacitors:

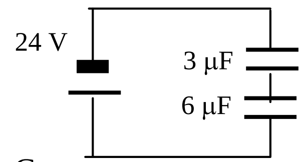
$Q_3 = Q_6 = 1200\ \mu C - 800\ \mu C$; $Q_6 = 400\ \mu C$

$$V_6 = \frac{Q_6}{C_6} = \frac{400\ \mu C}{6\ \mu F};$$ **$V_6 = 66.7\ V$**

You should show that $V_3 = 133.3$ V, so that $V_3 + V_6 = 200$ V.

*26-27. A 6-μF and a 3-μF capacitor are connected in series with a 24-V battery. If they are connected in series, what are the charge and voltage across each capacitor?

$$C = \frac{C_6 C_3}{C_6 + C_3} = \frac{(6\mu\text{F})(3\mu\text{F})}{6\mu\text{F} + 3\mu\text{F}}; \qquad C_e = 2.00\ \mu\text{F}$$



$Q_T = C_e V$ = (2 μF)(24 V) = 48 μF; $Q_3 = Q_6 = Q_T$ = 48 μC

$$V_3 = \frac{Q_3}{C_3} = \frac{48\ \mu\text{C}}{3\ \mu\text{F}};$$

V_3 = 16.0 V, Q_3 = 48.0 μC

$$V_6 = \frac{Q_6}{C_6} = \frac{48\ \mu\text{C}}{6\ \mu\text{F}};$$

V_6 = 8.00 V, Q_6 = 48.0 μC

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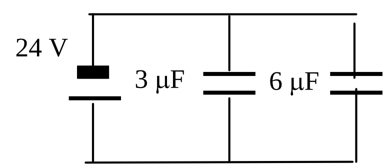
*26-28. If the 6 and 3-μF capacitors of Problem 26-27 are reconnected in parallel with a 24-V battery, what are the charge and voltage across each capacitor?



C_e = 3 μF + 6 μF; C_e = 9 μF

Q_T = (9 μF)(24 V) = 216 μC; **$V_3 = V_6$ = 24 V**

Q_3 = (3 μF)(24 V); Q_6 = (6 μF)(24 V); **Q_3 = 72 μC; Q_6 = 144 μC**

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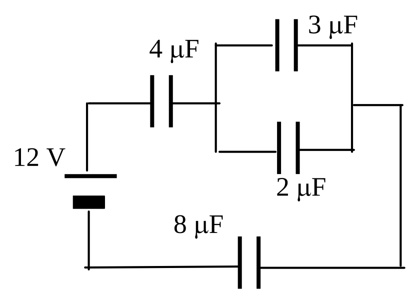
*26-29. Compute the equivalent capacitance for the entire circuit shown in Fig. 26-16. What is the total charge on the equivalent capacitance?

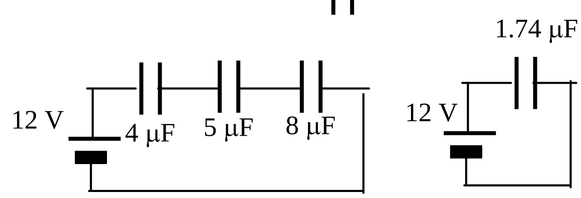


$C_{2,3}$ = 2 μF + 3 μF = 5 μF

$$\frac{1}{C_e} = \frac{1}{4\ \mu\text{F}} + \frac{1}{5\ \mu\text{F}} + \frac{1}{8\ \mu\text{F}};$$

C_e = 1.74 μF

$Q_T = C_e V$ = 20.9 μC



Also: $Q_4 = Q_5 = Q_8$ = 20.9 μC

*26-30. What are the charge and voltage across each capacitor of Fig. 26-16? (*See Prob. 26-29.)*

$$Q_4 = Q_5 = Q_8 = 20.9\ \mu\text{C};\quad V_4 = \frac{Q_4}{C_4} = \frac{20.9\ \mu\text{C}}{4\ \mu\text{F}} = 5.22\text{ V};\quad V_8 = \frac{Q_8}{C_8} = \frac{20.9\ \mu\text{C}}{8\ \mu\text{F}} = 2.61\text{ V}$$

$$V_5 = V_3 = V_2 = \frac{Q_5}{C_5} = \frac{20.9\ \mu\text{C}}{5\ \mu\text{F}} = 4.17\text{ V};\quad Q_3 = C_3V_3 = (3\ \mu\text{F})(4.17\text{ V}) = 12.5\ \mu\text{C}$$

$Q_2 = (2\ \mu\text{F})(4.17\text{ V}) = 8.35\ \mu\text{C}$; *Note that* $Q_2 + Q_3 = Q_5 = Q_T = 20.9\ \mu\text{C}$

Answer summary: $Q_2 = 8.35\ \mu\text{C}$; $Q_3 = 12.5$ C, $Q_4 = Q_8 = 20.9$ C

$V_2 = V_3 = 4.17$ V; $V_4 = 5.22$ V; $V_8 = 2.61$ V

The Energy of a Charged Capacitor

26-31. What is the potential energy stored in the electric field of a 200-μF capacitor when it is charged to a voltage of 2400 V?

$$P.E. = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(200 \times 10^{-6}\text{ F})(2400\text{ V})^2;\quad P.E. = 576\text{ J}$$

26-32. What is the energy stored on a 25-μF capacitor when the charge on each plate is 2400 μF? What is the voltage across the capacitor?

$$P.E. = \frac{Q^2}{2C} = \frac{(2400 \times 10^{-6}\text{C})^2}{2(25 \times 10^{-6}\text{ F})};\quad \boxed{P.E. = 115\text{ mJ}}$$

$$V = \frac{Q}{C} = \frac{2400\ \mu\text{C}}{25\ \mu\text{F}};\quad \boxed{V = 96.0\text{ V}}$$

26-33 How much work is required to charge a capacitor to a potential difference of 30 kV if 800 μC are on each plate?

$$Work = P.E. = \tfrac{1}{2}QV;\quad Work = \tfrac{1}{2}(800 \times 10^{-6}\text{ C})(30 \times 10^{3}\text{ V})$$

$$\boxed{Work = 12.0\text{ J}}$$

*26-34. A parallel plate capacitor has a plate area of 4 cm^2 and a separation of 2 mm. A dielectric of constant K = 4.3 is placed between the plates, and the capacitor is connected to a 100-V battery. How much energy is stored in the capacitor?

$$C = K\varepsilon_0 \frac{A}{d} = \frac{(4.3)(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(4 \text{ x } 10^{-4}\text{ m}^2)}{0.002 \text{ m}}; \quad C = 7.61 \text{ pF}$$

$$P.E. = ½CV^2 = ½(7.61 \text{ x } 10^{-12}\text{ F})(100 \text{ V})^2; \quad \boxed{P.E. = 3.81 \text{ x } 10^{-8}\text{ J}}$$

Challenge Problems

26-35. What is the break-down voltage for a capacitor with a dielectric of glass (K = 7.5) if the plate separation is 4 mm? The average dielectric strength is 118 MV/m.

$$V = E_m d = (118 \text{ x } 10^6 \text{ V})(0.004 \text{ m}); \quad \boxed{V = 472 \text{ kV}}$$

26-36. A capacitor has a potential difference of 240 V, a plate area of 5 cm^2 and a plate separation of 3 mm. What is the capacitance and the electric field between the plates? What is the charge on each plate?

$$C = \varepsilon_0 \frac{A}{d} = \frac{(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(5 \text{ x } 10^{-4}\text{m}^2)}{0.003 \text{ m}}; \quad \boxed{C = 1.48 \text{ pF}}$$

$$E = \frac{240 \text{ V}}{0.003 \text{ m}}; \quad \boxed{E = 8.00 \text{ x } 10^4\text{ V/m}} \quad Q = (1.48 \text{ x } 10^{-12}\text{ F})(240 \text{ V}); \quad \boxed{Q = 0.355 \text{ nC}}$$

26-37. Suppose the capacitor of Problem 26-34 is disconnected from the 240-V battery and then mica (K = 5) is inserted between the plates? What are the new voltage and electric field? If the 240-battery is reconnected, what charge will be on the plates?

$$K = \frac{V_0}{V} = \frac{E_0}{E}; \quad V = \frac{240 \text{ V}}{5} = \boxed{48.0 \text{ V}}; \quad E = - \ E = \frac{80{,}000 \text{ V/m}}{5} = \boxed{1.60 \text{ x } 10^4\text{ V/m}}$$

$$C = KC_o = 5(1.48 \text{ pF}) = 7.40 \text{ pF}; \quad \text{Q = CV} = (7.40 \text{ pF})(240 \text{ V}) = \boxed{1.78 \text{ nC}}$$

26-38. A 6-μF capacitor is charged with a 24-V battery and then disconnected. When a dielectric is inserted, the voltage drops to 6 V. What is the total charge on the capacitor after the battery has been reconnected?

$$K = \frac{V_o}{V} = \frac{24\text{ V}}{6\text{ V}} = 4; \quad C = \text{ K}C_o = 4(6\ \mu\text{F}) = 24\mu\text{F};$$

$$Q = CV = (24\ \mu\text{F})(24\text{ V}); \quad \boxed{Q = 576\ \mu\text{C}}$$

26-39. A capacitor is formed from 30 parallel plates 20 x 20 cm. If each plate is separated by 2 mm of dry air, what is the total capacitance?

Thirty stacked plates means that there are 29 spaces, which make for 29 capacitors:

$$C = 29\varepsilon_0 \frac{A}{d} = \frac{(29)(8.85\text{ x }10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.2\text{ m})^2}{0.002\text{ m}}; \quad \boxed{C = 5.13\text{ nF}}$$

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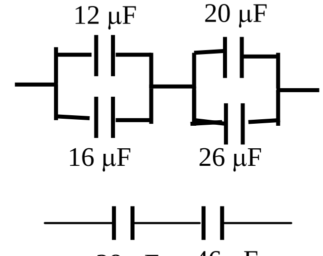
*26-40. Four capacitors A, B, C, and D have capacitances of 12, 16, 20, and 26 μF, respectively. Capacitors A and B are connected in parallel. The combination is then connected in series with C and D. What is the effective capacitance?



C_1 = 12 μF + 16 μF = 26 F; C_2 = 28 μF + 26 μF = 46 μF

$$C_e = \frac{C_1C_2}{C_1 + C_2} = \frac{(28\ \mu\text{F})(46\ \mu\text{F})}{28\ \mu\text{F} + 46\ \mu\text{F}};$$

$$\boxed{C_e = 17.4\ \mu\text{F}}$$

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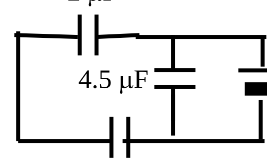
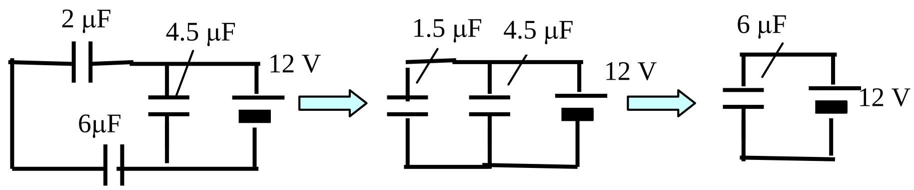
*26-41. Consider the circuit drawn in Fig. 32-17. What is the equivalent capacitance of the circuit? What are the charge and voltage across the 2-μF capacitor? *(Redraw Fig.)*



$$\frac{1}{C_{6,2}} = \frac{1}{2\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}; \qquad C_{6,2} = 1.50\ \mu\text{F}$$

$C_{6,2}$ = 1.5 μF + 4.5 μF = 6 μF C_e 6 μF

*26-41. (Cont.)



$Q_T = (6\ \mu F)(12\ V) = 72\ \mu C$; $Q_{1.5} = Q_2 = Q_3 = (1.5\ \mu F)(12\ V) = 18\ \mu C$

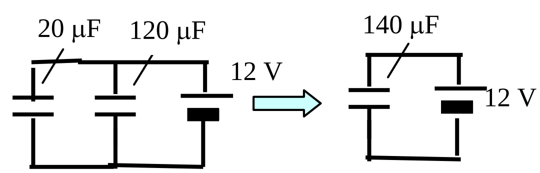
$$V_2 = \frac{Q_2}{C_2} = \frac{18 \mu C}{2\ \mu F} = 9.00\ V; \quad \boxed{C_e = 6.00\ \mu F, \quad V_2 = 9.00\ V; \quad Q_2 = 18\ \mu C}$$

---

*26-42. Two identical 20-μF capacitors A and B are connected in parallel with a 12-V battery. What is the charge on each capacitor if a sheet of porcelain (K = 6) is inserted between the plates of capacitor B and the battery remains connected?

$C_A = 20\ \mu F$; $C_B = 6(20\ \mu F) = 120\ \mu F$

$C_e = 120\ \mu F + 20\ \mu F$; $C_e = 140\ \mu F$

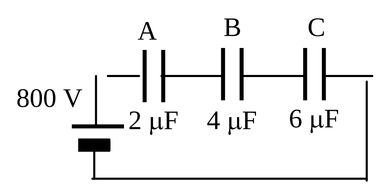


$Q_B = (120\ \mu F)(12\ V) = \boxed{1440\ \mu C;}$ $Q_A = (20\ \mu F)(12\ V) = \boxed{240\ \mu C}$

Show that BEFORE insertion of dielectric, the charge on EACH was 240 μC!

---

*26-43. Three capacitors A, B, and C have respective capacitances of 2, 4, and 6 μF. Compute the equivalent capacitance if they are connected in series with an 800-V source. What are the charge and voltage across the 4-μF capacitor?

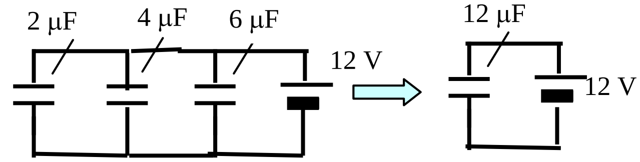


$$\frac{1}{C_e} = \frac{1}{2\ \mu F} + \frac{1}{4\ \mu F} + \frac{1}{6\ \mu F}; \quad \boxed{C_e = 1.09\ \mu F}$$

$Q_T = C_e V = (1.09\ \mu F)(800\ V)$; $Q_T = 873\ \mu C$; $Q_2 = Q_4 = Q_6 = 873\ \mu C$

$$\textit{Also:}; \quad V_4 = \frac{873\ \mu C}{4\ \mu F} = 218\ V; \quad \boxed{Q_4 = 873\ \mu C; \quad V_4 = 218\ V}$$

*26-44. Suppose the capacitors of Problem 26-41 are reconnected in parallel with the 800-V source. What is the equivalent capacitance? What are the charge and voltage across the 4-μF capacitor?



C_e = 2 μF + 4 μF + 6 μF; **C_e = 12 μF**

$Q_T = C_eV$ = (12 μF)(12 V) = 144 μC

$V_T = V_2 = V_3 = V_4$ = 12 V; $Q_4 = C_4V_4$ = (4 μF)(12 V); **Q_4 = 48 μC; V_4 = 12 V**

---

*26-45. Show that the total capacitance of a multiple-plate capacitor containing *N* plates separated by air is given by:

$$C_0 = \frac{(N-1)\varepsilon_0 A}{d}$$

where *A* is the area of each plate and *d* is the separation of each plate.

For a multiplate capacitor, if there are a total of N plates, we have the equivalent of (N – 1) capacitors each of area A and separation d. Hence, the above equation.

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*26-46. The energy density, *u*, of a capacitor is defined as the energy (*P.E.)* per unit volume (*Ad*) of the space between the plates. Using this definition and several formulas from this chapter, derive the following relationship for finding the energy density, u:

$$u = \frac{1}{2}\varepsilon_0 E^2$$

where *E* is the electric field intensity between the plates?

$$P.E. = ½CV^2; \quad V = Ed; \quad C = \varepsilon_0 \frac{A}{d}; \quad u = \frac{P.E.}{Ad}$$

$$u = \frac{½CV^2}{Ad} = \frac{½\left(\varepsilon_0 \frac{A}{d}\right)\left(E^2d^2\right)}{Ad}; \qquad \boxed{u = ½\varepsilon_0 E^2}$$

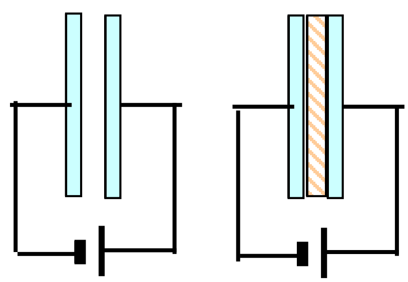
*26-47. A capacitor with a plate separation of 3.4 mm is connected to a 500-V battery. Use the relation derived in Problem 26-44 to calculated the energy density between the plates?

$$E = \frac{V}{d} = \frac{500\text{ V}}{0.0034\text{ m}} = 1.47 \times 10^5\text{ V/m}; \qquad u = \tfrac{1}{2}\varepsilon_o E^2$$

$$u = \tfrac{1}{2}(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(1.47 \times 10^4\text{ V/m})^2; \quad \boxed{u = 95.7\text{ mJ/m}^3}$$

Critical Thinking Problems

26-48. A certain capacitor has a capacitance of 12 μF when its plates are separated by 0.3 mm of vacant space. A 400-V battery charges the plates and is then disconnected from the capacitor. (a) What is the potential difference across the plates if a sheet of Bakelite (K = 7) is inserted between the plates? (b) What is the total charge on the plates? (c) What is the capacitance with the dielectric inserted? (d) What is the permittivity of Bakelite? (d) How much additional charge can be placed on the capacitor if the 400-V battery is reconnected?



$$E_0 = \frac{V_0}{d} = \frac{400\text{ V}}{0.0003\text{ m}}; \quad E_0 = 1.33 \times 10^6\text{ V/m}$$

$$C = KC_0 = (7)(12\ \mu\text{F}) = 84\ \mu\text{F}$$

(a) $K = \frac{V_0}{V}; \quad V = \frac{400\text{ V}}{7}; \quad \boxed{V = 57.1\text{ V}}$

(b) $Q_0 = C_0V_0 = (12\ \mu\text{F})(400\text{ V}); \quad \boxed{Q_0 = 4800\ \mu\text{C}}$ *(disconnected)* $\boxed{\text{(c) } 48\ \mu\text{F}}$

(d) $\varepsilon = K\varepsilon_o = (7)(8.85 \times 10^{-12}\text{ N m}^2/\text{C}^2); \quad \boxed{\varepsilon = 6.20 \times 10^{-11}\text{ N m}^2/\text{C}^2}$

(e) $Q = CV = (84\ \mu\text{F})(400\text{ V}); \quad Q = 33.6\text{ mC}$ *(400-V Battery reconnected)*

The added charge is: $Q - Q_0 = 33.6\text{ mC} - 4.8\text{ mC}; \quad \boxed{\Delta Q = 28.8\text{ mC}}$

*****26-49. A medical defibrillator uses a capacitor to revive heart attack victims. Assume that a 65-μF capacitor in such a device is charged to 5000 V. What is the total energy stored? If 25 percent of this energy passes through a victim in 3 ms, what power is delivered?

$$P.E. = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(65 \times 10^{-6}\ \text{F})(5000\ \text{V}); \quad \boxed{P.E. = 162\ \text{mJ}}$$

$$Power = \frac{0.25(0.162\ \text{J})}{0.003\ \text{s}} = 13.5\ \text{W}; \quad \boxed{Power = 13.5\ \text{W}}$$

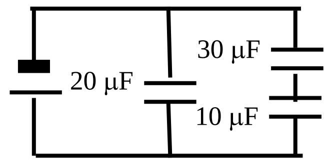
*****26-50. Consider three capacitors of 10, 20, and 30 μF. Show how these might be connected to produce the maximum and minimum equivalent capacitances and list the values. Draw a diagram of a connection that would result in an equivalent capacitance of 27.5 μF. Show a connection that will result in a combined capacitance of 22.0 μF.

The maximum is for a parallel connection: $C = 10\ \mu\text{F} + 20\ \mu\text{F} + 30\ \mu\text{F}$; $\boxed{C_{max} = 60\ \mu\text{F}}$

Minimum is for series: $$\frac{1}{C_e} = \frac{1}{20\ \mu\text{F}} + \frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}; \quad \boxed{C_{min} = 5.45\ \mu\text{F}}$$

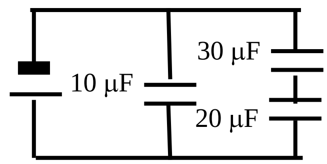
<u>*Case 1:*</u> $$\frac{1}{C_1} = \frac{1}{10\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}; \quad C_1 = 7.5\ \mu\text{F}$$

$$C_e = C_1 + 20\ \mu\text{F} = 12\ \mu\text{F} + 20\mu\text{F}; \quad \boxed{C_e = 22.0\ \mu\text{F}}$$

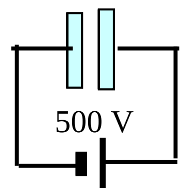


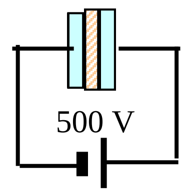
<u>*Case 2:*</u> $$\frac{1}{C_2} = \frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}; \quad C_1 = 12\ \mu\text{F}$$

$$C_e = C_1 + 20\ \mu\text{F} = 12\ \mu\text{F} + 20\mu\text{F}; \quad \boxed{C_e = 22.0\ \mu\text{F}}$$



*****26-51. A 4-μF air capacitor is connected to a 500-V source of potential difference. The capacitor is then disconnected from the source and a sheet of mica (K = 5) is inserted between the plates? What is the new voltage on the capacitor? Now reconnect the 500-V battery and calculate the final charge on the capacitor? By what percentage did the total energy on the capacitor increase due to the dielectric?





(a) $K = \dfrac{V_0}{V}$; $V = \dfrac{500\text{ V}}{5}$; $\boxed{V = 100\text{ V}}$

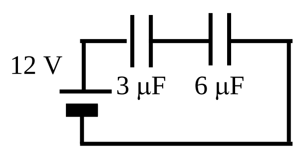
(b) $Q_0 = C_0V_0$ = (4 μF)(500 V); $\boxed{Q_0 = 2000\ \mu\text{C}}$

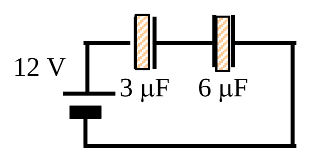
(c) $(P.E.)_0 = ½C_0V^2 = ½(4 \times 10^{-6}\text{ F})(500\text{ V})^2$; $(P.E.)_0$ = 0.500 J; C = 5(4 μF) = 20 μF

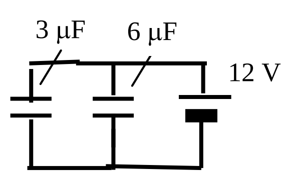
$(P.E.) = ½CV^2 = ½(20 \times 10^{-6}\text{ F})(500\text{ V})^2$; $(P.E.)_0$ = 2.50 J

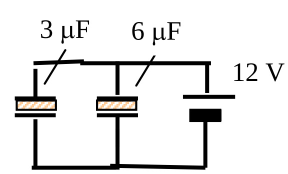
$$Percent\ increase = \frac{2.50\text{ J} - 0.50\text{ J}}{0.500\text{ J}};\quad \boxed{\text{Percent increase} = 400\%}$$

*****26-52. A 3-μF capacitor and a 6-μF capacitor are connected in series with a 12-V battery. What is the total stored energy of the system? What is the total energy if they are connected in parallel? What is the total energy for each of these connections if mica (K = 5) is used as a dielectric for each capacitor?









Series: $C_e = \dfrac{(3\ \mu\text{F})(6\ \mu\text{F})}{3\ \mu\text{F} + 6\ \mu\text{F}} = 2.00\ \mu\text{F}$

$P.E. = ½(2 \times 10^{-6}\text{ F})(12\text{ V})^2 =$ $\boxed{0.144\text{ mJ}}$

Parallel: C_e = 3 μF + 6 μF = 9 μF

$P.E. = ½(9 \times 10^{-6}\text{ F})(12\text{ V})^2 =$ $\boxed{0.648\text{ mJ}}$ C_3 = 5(3 μF) = 15 μF; C_6 = 5(6 μF) = 30 μF

Series: $C_e = \dfrac{(15\ \mu\text{F})(30\ \mu\text{F})}{15\ \mu\text{F} + 30\ \mu\text{F}} = 10.00\ \mu\text{F}$; Parallel: C = 15 μF + 30 μF = 45 μF

$P.E. = ½(10 \times 10^{-6}\text{ F})(12\text{ V})^2 =$ $\boxed{0.720\text{ mJ}}$; $P.E. = ½(45 \times 10^{-6}\text{ F})(12\text{ V})^2 =$ $\boxed{3.24\text{ mJ}}$