Chapter 24. The Electric Field

The Electric Field Intensity

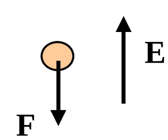
24-1. A charge of +2 μC placed at a point P in an electric field experiences a downward force of 8×10^{-4} N. What is the electric field intensity at that point?

$$E = \frac{F}{q} = \frac{8 \times 10^{-4}\text{ N}}{2 \times 10^{-6}\text{ C}};\quad \boxed{E = 400\text{ N/C, } \textit{downward}}$$

24-2. A –5 nC charge is placed at point P in Problem 24-1. What are the magnitude and direction of the force on the –5 nC charge? (*Direction of force F is opposite field E*)

$$F = qE = (-5 \times 10^{-9}\text{ C})(-400\text{ N/C});\quad \boxed{F = 2.00 \times 10^{-6}\text{ N, } \textit{upward}}$$

24-3. A charge of –3 μC placed at point A experiences a downward force of 6×10^{-5} N. What is the electric field intensity at point A?



A negative charge will experience a force opposite to the field.

Thus, if the –3 μC *charge has a downward force, the* **E** *is upward.*

$$E = \frac{F}{q} = \frac{-6 \times 10^{-5}\text{ N}}{-3 \times 10^{-6}\text{ C}};\quad \boxed{E = 20\text{ N/C, } \textit{upward}}$$

24-4. At a certain point, the electric field intensity is 40 N/C, due east. An unknown charge, receives a westward force of 5×10^{-5} N. What is the nature and magnitude of the charge?

If the force on the charge is opposite the field E, it must be a negative charge.

$$E = \frac{F}{q};\quad q = \frac{F}{E} = \frac{-5 \times 10^{-5}\text{ N}}{40\text{ N/C}};\quad \boxed{q = -1.25\ \mu\text{C}}$$

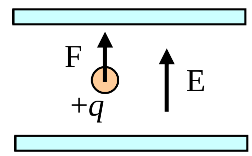
24-5. What are the magnitude and direction of the force that would act on an electron ($q = -1.6 \times 10^{-19}$ C) if it were placed at (a) point P in Problem 24-1? (b) point A in Problem 24-3?

The electric force on an electron will always be opposite the electric field.

(a) $F = qE = (-1.6 \times 10^{-19}\text{ C})(-400\text{ N/C})$; $\boxed{F = 6.40 \times 10^{-17}\text{ N, } upward}$

(b) $F = qE = (-1.6 \times 10^{-19}\text{ C})(+20\text{ N/C})$; $\boxed{F = -3.20 \times 10^{-18}\text{ N, } downward}$

24-6. What must be the magnitude and direction of the electric field intensity between two horizontal plates if one wants to produce an upward force of 6×10^{-4} N on a +60-μC charge? (*The upward force on +q means E is also upward.)*



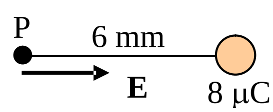
$$E = \frac{F}{q} = \frac{6 \times 10^{-4}\text{N}}{60 \times 10^{-6}\text{C}};\quad \boxed{E = 10.0\text{ N/C, } up}$$

24-7. The uniform electric field between two horizontal plates is 8×10^4 C. The top plate is positively charged and the lower plate has an equal negative charge. What are the magnitude and direction of the electric force acting on an electron as it passes horizontally through the plates? (*The electric field is from + to -, i.e., downward; force on e is up.)*

$$F = qE = (-1.6 \times 10^{-19}\text{ C})(8 \times 10^4\text{ N/C});\quad \boxed{F = 1.28 \times 10^{-14}\text{ N, } upward}$$

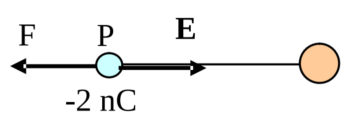
24-8. Find the electric field intensity at a point P, located 6 mm to the left of an 8-μC charge. What are the magnitude and direction of the force on a –2-nC charge placed at point P?

$$E = \frac{kQ}{r^2} = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(8 \times 10^{-6}\text{C})}{(6 \times 10^{-3}\text{mm})^2};$$



$$\boxed{E = 2.00 \times 10^9\text{ N/C, toward Q}}$$

$$F = qE = (-2 \times 10^{-9}\text{ C})(2.00 \times 10^9\text{ N/C})$$



$$\boxed{F = -4.00\text{ N, away from Q}}$$

24-9. Determine the electric field intensity at a point P, located 4 cm above a –12-μC charge.

What are the magnitude and direction of the force on a +3-nC charge placed at point P?

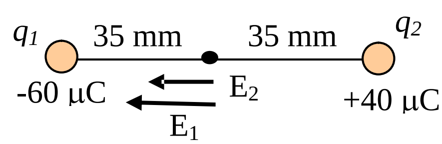
Electric field will be downward, since that is the direction a positive charge would move.

$$E = \frac{kQ}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(-12 \text{ x } 10^{-6}\text{C})}{(0.04 \text{ m})^2};$$ E = -6.75 x 10^7 N/C, *downward*

$F = qE$ = (3 x 10^{-9} C)(-6.75 x 10^7 N/C) ; F = -0.202 N, *downward*

Calculating the Resultant Electric Field Intensity

24-10. Determine the electric filed intensity at the midpoint of a 70 mm line joining a –60-μC charge with a +40-μC charge.



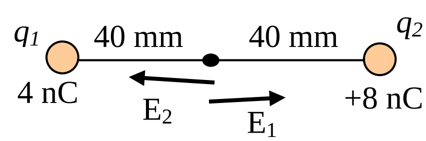
$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(-60 \text{ x } 10^{-6}\text{C})}{(0.035 \text{ m})^2}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(40 \text{ x } 10^{-6}\text{C})}{(0.035 \text{ m})^2};$$

$\mathbf{E_R = E_1 + E_2}$ *(Both to left)*

E_R = -4.41 x 10^8 N/C – 2.94 x 10^8 N/C ; $\mathbf{E_R}$ = 7.35 x 10^8 N/C. *toward* –60 μC

24-11. An 8-nC charge is located 80 mm to the right of a +4-nC charge. Determine the field intensity at the midpoint of a line joining the two charges.



$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(4 \text{ x } 10^{-9}\text{C})}{(0.040 \text{ m})^2}$$

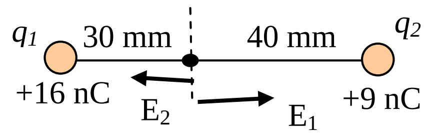
$$E_2 = \frac{kq_2}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(8 \text{ x } 10^{-9}\text{C})}{(0.040 \text{ m})^2};$$

$\mathbf{E_R = E_1 + E_2}$ *(E_1 right, E_2 left)*

E_R = -4.50 x 10^4 N/C + 2.25 x 10^4 N/C ; $\mathbf{E_R}$ = -2.25 x 10^4 N/C, *left*

Note: The directions of the E field are based on how a test + charge would move.

24-12. Find the electric field intensity at a point 30 mm to the right of a 16-nC charge and 40 mm to the left of a 9-nC charge.

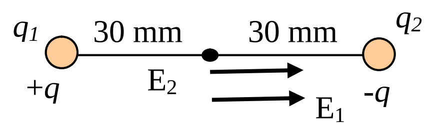


$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(16 \text{ x } 10^{-9}\text{C})}{(0.030 \text{ m})^2}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(9 \text{ x } 10^{-9}\text{C})}{(0.040 \text{ m})^2};$$ **E$_R$ = E$_1$ + E$_2$** *(E_1 right, E_2 left)*

E_R = 16.0 x 10^4 N/C - 5.06 x 10^4 N/C ; **E$_R$** = 1.09 x 10^5 N/C, *right*

24-13. Two equal charges of opposite signs are separated by a horizontal distance of 60 mm. If the resultant electric field at the midpoint of the line is 4 x 10^4 N/C. What is the magnitude of each charge?



Equal and opposite charges make field at center equal to vector sum with both to left or both to right.. **E$_R$ = E$_1$ + E$_2$ =** $E_1 + E_2$

$$E = \frac{2kq}{r^2} = 4 \text{ x } 10^4\text{N/C}; \quad \frac{2(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)q}{(0.030 \text{ m})^2} = 4 \text{ x } 10^4\text{N/C}$$

q = 2.00 nC *(One positive and the other negative.)*

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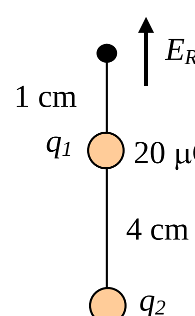
*24-14. A 20-μC charge is 4 cm above an unknown charge q. The resultant electric intensity at a point 1 cm above the 20-μC charge is 2.20 x 10^9 N/C and is directed upward? What are the magnitude and sign of the unknown charge?



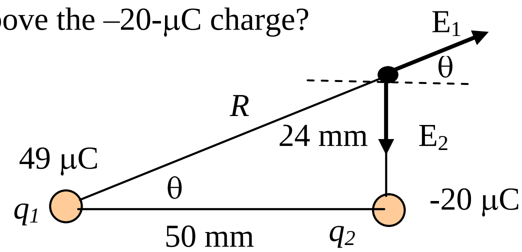
E$_1$ + E$_2$ = 2.20 x 10^9 N/C; *First we find E_1 and E_2*

$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(20 \text{ x } 10^{-6}\text{C})}{(0.010 \text{ m})^2}; \quad E_1 = 1.80 \text{ x } 10^9 \text{ N}$$

$E_2 = E_R - E_1$ = 2.20 x 10^9 N/C – 1.80 x 10^9 N/C; E_2 = 4 x 10^8 N/C, *up*

$$E_2 = \frac{kq_2}{r^2}; \quad q_2 = \frac{E_2 r^2}{k} = \frac{(4 \text{ x } 10^8\text{N/C})(0.05 \text{ m})^2}{(9 \text{ x } 10^9\text{N}\cdot\text{m}^2/\text{C}^2)};$$ $q = q_2$ = 111 μC

*24-15. A charge of –20 μC is placed 50 mm to the right of a 49μC charge. What is the resultant field intensity at a point located 24 mm directly above the –20-μC charge?



$$R = \sqrt{(50\text{ mm})^2 + (24\text{ mm})^2} = 55.5\text{ mm}$$

$$\tan\theta = \frac{24\text{ mm}}{50\text{ mm}}; \qquad \theta = 25.6^0$$

$$E_1 = \frac{kq_1}{r^2} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(49\text{ x }10^{-6}\text{C})}{(0.0555\text{ m})^2}; \qquad E_1 = 1.432\text{ x }10^8\text{ N/C at }25.6^0\text{ N of E}$$

$$E_2 = \frac{kq_1}{r^2} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(20\text{ x }10^{-6}\text{C})}{(0.024\text{ m})^2}; \qquad E_2 = 3.125\text{ x }10^8\text{ N/C, } \textit{downward}$$

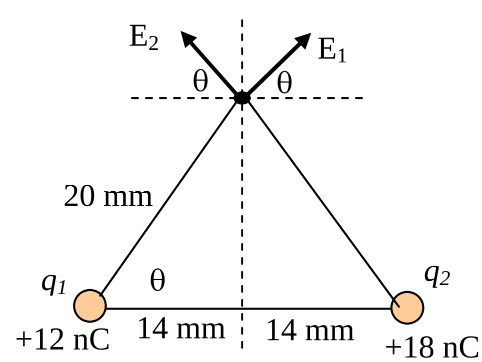
$$E_x = (1.432\text{ x }10^8\text{ N/C})\cos 25.6^0 + 0; \quad E_x = 1.291\text{ x }10^8\text{ N/C}$$

$$E_y = (1.432\text{ x }10^8\text{ N/C})\sin 25.6^0 - 3.125\text{ x }10^8\text{ N/C}; \quad E_y = -2.506\text{ x }10^8\text{ N/C}$$

$$E_R = \sqrt{(1.29\text{ x }10^8)^2 + (-2.51\text{ x }10^8)^2}; \quad E_R = 2.82\text{ x }10^8\text{ N/C}$$

$$\tan\theta = \frac{-2.51\text{ x }10^8\text{N/C}}{1.29\text{ x }10^8\text{N/C}}; \qquad \theta = 62.7^0\text{ S of E}; \qquad \boxed{E_R = 2.82\text{ x }10^8\text{ N/C, }297.3^0.}$$

*****24-16. Two charges of +12 nC and +18 nC are separated horizontally by 28 mm. What is the resultant field intensity at a point 20 mm from each charge and above a line joining the two charges?



$$\cos\theta = \frac{14\text{ mm}}{20\text{ mm}}; \qquad \theta = 45.6^0$$

$$E_1 = \frac{kq_1}{r^2} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(12\text{ x }10^{-9}\text{C})}{(0.020\text{ m})^2}$$

$E_1 = 2.70\text{ x }10^5$ N/C, 45.6^0 N of E

$$E_2 = \frac{kq_1}{r^2} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(18\text{ x }10^{-9}\text{C})}{(0.020\text{ m})^2}; \quad E_2 = 4.05\text{ x }10^5\text{ N/C, }45.6^0\text{ N of W}$$

*24-16. (Cont.) $E_x = (2.70 \times 10^5 \text{ N/C}) \cos 45.6^0 - (4.05 \times 10^5 \text{ N/C}) \cos 45.6^0 = -9.45 \times 10^4 \text{ N/C}$

$E_y = (2.70 \times 10^5 \text{ N/C}) \sin 45.6^0 - (4.05 \times 10^5 \text{ N/C}) \sin 45.6^0 = +4.82 \times 10^5 \text{ N/C}$

$$E_R = \sqrt{(-9.45 \times 10^4)^2 + (4.82 \times 10^5)^2}\ ; \quad E_R = 4.91 \times 10^5 \text{ N/C}$$

$$\tan\theta = \frac{4.82 \times 10^5 \text{ N/C}}{-9.45 \times 10^4 \text{ N/C}}; \quad \theta = 78.9^0 \text{ N of W}; \quad \boxed{E_R = 4.91 \times 10^5 \text{ N/C}, 101.1^0}$$

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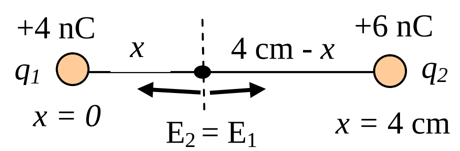
*24-17. A +4 nC charge is placed at x = 0 and a +6 nC charge is placed at x = 4 cm on an x-axis.

Find the point where the resultant electric field intensity will be zero?

$$E_1 = E_2; \quad \frac{kq_1}{x^2} = \frac{kq_2}{(4 \text{ cm - x})^2}$$



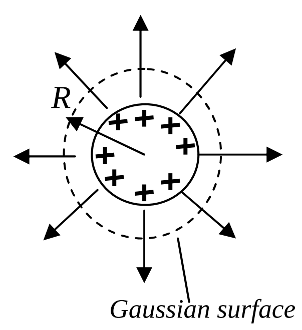
$$(4-x)^2 = \frac{q_2}{q_1}x^2 \quad \text{or} \quad 4-x = \sqrt{\frac{q_2}{q_1}}x$$

$$4 \text{ cm - x} = \sqrt{\frac{6 \text{ nC}}{4 \text{ nC}}}x; \quad 4 \text{ cm - x} = 1.225\,x; \quad \boxed{x = 1.80 \text{ cm}}$$

Applications of Gauss's Law

24-18. Use Gauss's law to show that the field outside a solid charged sphere at a distance *r* from its center is given by

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$



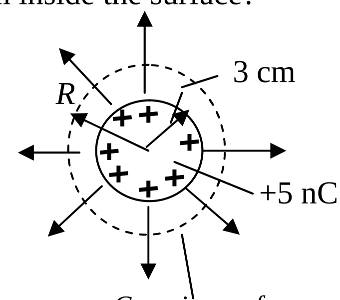
where Q is the total charge on the sphere.

Construct a spherical gaussian surface around the charged sphere at the distance r from its center. Then, we have

$$\Sigma\varepsilon_0 AE = \Sigma q\ ; \qquad \varepsilon_0 E(4\pi R^2) = Q$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 R^2}}$$

24-19. A charge of +5 nC is placed on the surface of a hollow metal sphere whose radius is 3 cm. Use Gauss's law to find the electric field intensity at a distance of 1 cm from the surface of the sphere? What is the electric field at a point 1 cm inside the surface?



Draw gaussian surface of radius R = 3 cm + 1 cm = 4 cm.

This surface encloses a net positive charge of +5 nC *and has a surface area of* $4\pi R^2$, *so Gauss' law gives us:*

(a) $\Sigma\varepsilon_0 AE = \Sigma q; \quad \varepsilon_0(4\pi R^2)E = q; \quad E = \dfrac{q}{4\pi\varepsilon_0 R^2}$

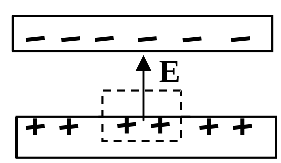
$$E = \frac{5 \text{ x } 10^{-9}\text{C}}{4\pi(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.04 \text{ m})^2};$$

E = 2.81 x 10^4 N/C, *radially outward.*

(b) *Draw a gaussian surface just <u>inside</u> the sphere. Now, <u>all</u> charge resides on the surface of the sphere, so that zero net charge is enclosed, and* $\Sigma\varepsilon_o AE = \Sigma q = 0$.

E = 0, inside sphere

24-20. Two parallel plates, each 2 cm wide and 4 cm long, are stacked vertically so that the field intensity between the two plates is 10,000 N/C directed upward. What is the charge on each plate? *First use Gauss' law to find E between plates.*



Draw gaussian cylinder of area A enclosing charge q.

$$\Sigma\varepsilon_0 AE = \Sigma q; \quad \varepsilon_0 AE = q; \quad E = \frac{q}{\varepsilon_0 A}$$

The charge density q/A enclosed is same as Q/A_p *for plate. First find q/A from E :*

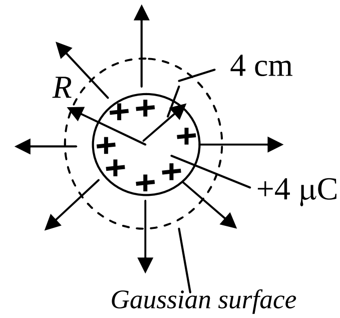
$$\frac{q}{A} = \varepsilon_0 E = (8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(10{,}000 \text{ N/C}); \quad \frac{q}{A} = 8.85 \text{ x } 10^{-8} \text{ C/m}^2$$

$$\frac{q}{A} = \frac{Q}{(0.02 \text{ m})(0.04 \text{ m})} = 8.85 \text{ x } 10^{-8} \text{ C/m}^2;$$

Q = 7.09 x 10^{-11} C

24-21. A sphere 8 cm in diameter has a charge of 4 μC placed on its surface. What is the electric field intensity at the surface, 2 cm outside the surface, and 2 cm inside the surface?

(a) *Draw gaussian surface just outside so that R* = 4 cm *and encloses the net charge of +4* uC. *Then,*



$$E = \frac{q_{net}}{4\pi\varepsilon_0 R^2} = \frac{4 \times 10^{-6}\text{C}}{4\pi(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.04 \text{ m})^2}$$

$E = 2.25 \times 10^7$ N/C, *radially outward*

(b) *Draw gaussian surface of radius R* = 4 cm + 2 cm = 6 cm. *This surface encloses a net positive charge of* +4 nC *and Gauss law gives:*

$$E = \frac{4 \times 10^{-6}\text{C}}{4\pi(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.06 \text{ m})^2};$$

$E = 9.99 \times 10^6$ N/C, *radially outward.*

(b) *Since no net charge is inside the surface,* $\Sigma\varepsilon_0 AE = \Sigma q = 0$.

E = 0, inside sphere

Challenge Problems

24-22. How far from a point charge of 90 nC will the field intensity be 500 N/C?

$$E = \frac{kQ}{r^2}; \quad r = \sqrt{\frac{kQ}{E}} = \sqrt{\frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(90 \times 10^{-9}\text{C})}{500 \text{ N/C}}};$$

r = 1.27 m

24-23. The electric field intensity at a point in space is found to be 5×10^5 N/C, directed due west. What are the magnitude and direction of the force on a –4-μC charge placed at that point?

Consider East positive: $F = qE = (-4\ \mu\text{C})(-5 \times 10^5 \text{ N/C})$;

F = 2.00 N, East

24-24. What are the magnitude and direction of the force on an alpha particle ($q = +3.2 \times 10^{-19}$ C) as it passes into an upward electric field of intensity 8×10^4 N/C? (*Choose up as +*)

$F = qE = (3.2 \times 10^{-19}\text{ C})(+8 \times 10^4)$; $\boxed{F = 2.56 \times 10^{-14}\text{ N}}$

24-25. What is the acceleration of an electron ($e = -1.6 \times 10^{-19}$ C) placed in a constant downward electric field of 4×10^5 N/C? What is the gravitational force on this charge if $m_e = 9.11 \times 10^{-31}$ kg. (*Choose up as +, then* $E = -4 \times 10^5$ N/C.)

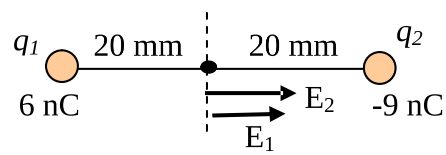
$F = qE = (-1.6 \times 10^{-19}\text{ C})(-4 \times 10^5\text{ N/C})$; $\boxed{F = 6.40 \times 10^{-14}\text{ N, upward}}$

$W = mg = (9.11 \times 10^{-31}\text{ kg})(9.8\text{ m/s}^2)$; $\boxed{W = 8.93 \times 10^{-30}\text{ N, downward}}$

The weight of an electron is often negligible in comparison with electric forces!

24-26. What is the electric field intensity at the midpoint of a 40 mm line between a 6-nC charge and a –9-nC charge? What force will act on a –2-nC charge placed at the midpoint?

$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(6 \times 10^{-9}\text{C})}{(0.020\text{ m})^2}$$



$$E_2 = \frac{kq_2}{r^2} = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(9 \times 10^{-9}\text{C})}{(0.020\text{ m})^2};$$

$\mathbf{E_R = E_1 + E_2}$ *(both to the right)*

$E_R = 1.35 \times 10^5\text{ N/C} + 2.025 \times 10^5\text{ N/C}$; $\boxed{\mathbf{E_R} = 3.38 \times 10^5\text{ N/C, right}}$

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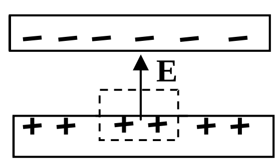
*24-27. The charge density on each of two parallel plates is 4 μC/m². What is the electric field intensity between the plates? *Recall that* $\sigma = q/A$, *and see Prob.24-20:*

$$\Sigma\varepsilon_0 AE = \Sigma q; \quad \varepsilon_0 AE = q; \quad E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

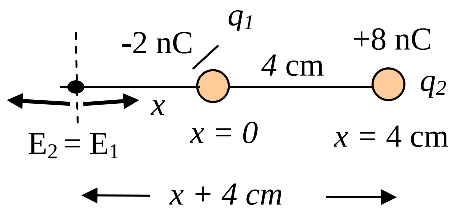


$$E = \frac{\sigma}{\varepsilon_0} = \frac{4 \times 10^{-6}\text{ C/m}^2}{(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)};$$ $\boxed{E = 4.52 \times 10^5\text{ N/C}}$

*24-28. A -2 nC charge is placed at x = 0 on the x-axis. A +8 nC charge is placed at x = 4 cm. At what point will the electric field intensity be equal to zero?

The point can only be to the left of the –2 nC

$$E_1 = E_2; \quad \frac{kq_1}{x^2} = \frac{kq_2}{(x + 4\text{ cm})^2}$$



$$(4+x)^2 = \frac{q_2}{q_1}x^2 \quad \text{or} \quad 4 + x = \sqrt{\frac{q_2}{q_1}}x$$

$$4\text{ cm} + x = \sqrt{\frac{8\text{ nC}}{2\text{ nC}}}x; \quad 4\text{ cm} + x = 2x; \quad x = 8.00\text{ cm}, \textit{ left, or } \boxed{x = -4.00\text{ cm}}$$

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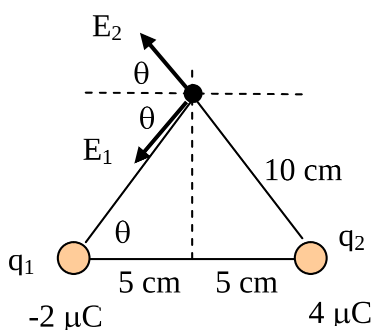
*24-29. Charges of –2 and +4 μC are placed at base corners of an equilateral triangle with 10-cm sides. What are the magnitude and direction of the electric field at the top corner?

$$\cos\theta = \frac{5\text{ mm}}{10\text{ mm}}; \qquad \theta = 60^0$$



$$E_1 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2 \text{ x } 10^{-6}\text{C})}{(0.10\text{ m})^2}$$

E_1 = 1.80 x 10^6 N/C, 60^0 N of E

$$E_2 = \frac{kq_1}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(4 \text{ x } 10^{-6}\text{C})}{(0.10\text{ m})^2}; \quad E_2 = 3.60 \text{ x } 10^6 \text{ N/C}, 60^0 \text{ N of W}$$

E_x = - (1.80 x 10^6 N/C) cos 60^0 – (3.60 x 10^6 N/C) cos 60^0 = -2.70 x 10^6 N/C

E_y = - (1.80 x 10^6 N/C) sin 60^0 + (3.60 x 10^6 N/C) sin 60^0 = +1.56 x 10^6 N/C

$$E_R = \sqrt{(-2.70 \text{ x } 10^6)^2 + (1.56 \text{ x } 10^6)^2}; \quad E_R = 3.12 \text{ x } 10^6 \text{ N/C}$$

$$\tan\theta = \frac{1.56 \text{ x } 10^6 \text{ N/C}}{-2.70 \text{ x } 10^6 \text{ N/C}}; \quad \theta = 30.0^0 \text{ N of W}; \quad \boxed{E_R = 3.12\text{x } 10^6 \text{ N/C}, 150.0^0}$$

24-30. What are the magnitude and direction of the force that would act on a –2-μC charge placed at the apex of the triangle described by Problem 24-29?

First we find the magnitude: $F = qE = (2 \times 10^{-6}\text{ C})(3.12 \times 10^{6}\text{ N/C})$; $F = 6.24$ N

Force is opposite field: $\theta = 180^0 + 150^0 = 330^0$ $\boxed{F = 6.24\text{ N}, 330^0}$

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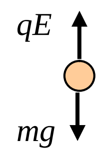
*24-31. A 20-mg particle is placed in a uniform downward field of 2000 N/C. How many excess electrons must be placed on the particle for the electric and gravitational forces to balance? (*The gravitational force must balance the electric force.)*



$$qE = mg; \quad q = \frac{mg}{E} = \frac{(2 \times 10^{-5}\text{ kg})(9.8\text{ m/s}^2)}{2000\text{ N/C}}$$

$$q = 9.00 \times 10^{-8}\text{ C}; \quad 1\text{ e} = 1.6 \times 10^{-19}\text{ C}$$

$$q_e = 9.8 \times 10^{-8}\text{C}\left(\frac{1\ e}{1.6 \times 10^{-19}\text{C}}\right); \quad \boxed{q_e = 6.12 \times 10^{11}\text{ electrons}}$$

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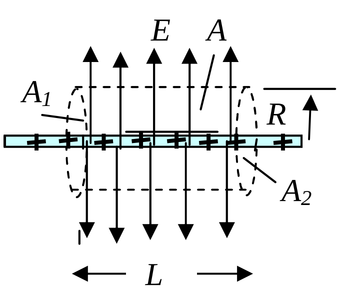
*24-32. Use Gauss's law to show that the electric field intensity at a distance R from an infinite line of charge is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$



where λ is the charge per unit length.

Gaussian surface area $A = [\,(2\pi R)L + A_1 + A_2\,]$

$$\Sigma\varepsilon_0 AE = \Sigma q; \quad \varepsilon_0 A_1 E_1 + \varepsilon_0 A_2 E_2 + \varepsilon_0(2\pi RL)E = q_{net}$$

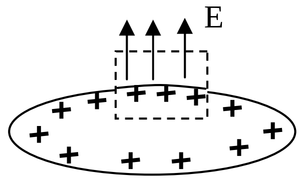
The fields E_1 and E_2 are balanced through ends: $\varepsilon_0(2\pi RL)E = q_{net}$;

$$E = \frac{q}{2\pi\varepsilon_0 RL}$$ *But the linear charge density is λ = q/L, therefore:*

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 R}}$$

*24-33. Use Gauss's law to show that the field just outside any solid conductor is given by

$$E = \frac{\sigma}{\varepsilon_0}$$



Draw a cylindrical pill box as gaussian surface.

The field lines through the sides are balanced and the field inside the surface is zero.

Thus, only one surface needs to be considered, the area A of the top of the pill box.

$$\sum \varepsilon_0 AE = \sum q; \qquad \varepsilon_o EA = q; \qquad E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}; \qquad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

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*24-34. What is the electric field intensity 2 m from the surface of a sphere 20 cm in diameter having a surface charge density of +8 nC/m^2? [A = 4πR^2; r = 2 m + 0.2 m = 2.2 m]

$$q = \sigma A = (8 \text{ x } 10^{-9} \text{ C})(4\pi)(0.20 \text{ m})^2; \qquad q = 2.01 \text{ x } 10^{-12} \text{ C}$$

$$E = \frac{kq}{r^2} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2.01 \text{ x } 10^{-12}\text{C})}{(2.20 \text{ m})^2}; \qquad E = 3.74 \text{ x } 10^{-3} \text{ N/C}$$

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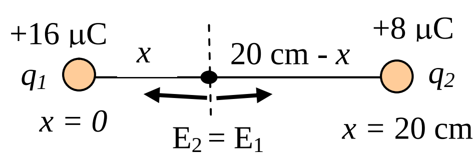
*24-35. A uniformly charged conducting sphere has a radius of 24 cm and a surface charge density of +16 μC/m^2. What is the total number of electric field lines leaving the sphere?

$$q = \sigma A = (16 \text{ x } 10^{-6} \text{ C})(4\pi)(0.24 \text{ m})^2; \qquad q = 1.16 \text{ x } 10^{-5} \text{ C}$$

$$N = \Sigma\varepsilon_o AE = q; \qquad \boxed{N = 1.16 \text{ x } 10^{-5} \text{ lines}}$$

*****24-36. Two charges of +16 μC and +8 μC are 200 mm apart in air. At what point on a line joining the two charges will the electric field be zero? (200 mm = 20 cm)

$$E_1 = E_2; \qquad \frac{kq_1}{x^2} = \frac{kq_2}{(20 \text{ cm - x})^2}$$



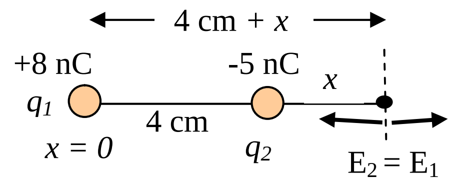
$$(20 - x)^2 = \frac{q_2}{q_1}x^2 \quad \text{or} \quad 20 - x = \sqrt{\frac{q_2}{q_1}}x$$

*24-36 (Cont.) $20\text{ cm} - x = \sqrt{\dfrac{8\ \mu\text{C}}{16\ \mu\text{C}}}\,x$; $20\text{ cm} - x = 0.707\,x$; **$x = 11.7$ cm**

*****24-37. Two charges of +8 nC and –5 nC are 40 mm apart in air. At what point on a line joining the two charges will the electric field intensity be zero?

The point can only be to right of –5 nC *charge*

$$E_2 = E_1; \quad \frac{kq_2}{x^2} = \frac{kq_1}{(x + 4\text{ cm})^2}$$



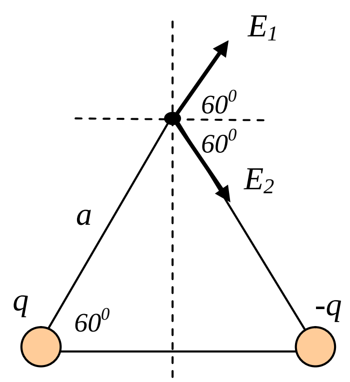
$$(4+x)^2 = \frac{q_1}{q_2}x^2 \quad \text{or} \quad 4 + x = \sqrt{\frac{q_1}{q_2}}\,x$$

$$4\text{ cm} + x = \sqrt{\frac{8\text{ nC}}{5\text{ nC}}}\,x; \quad 4\text{ cm} + x = 1.265\,x;$$

$x = 15.1$ cm outside of –5 nC charge.

Critical Thinking Questions

*****24-38. Two equal but opposite charges +*q* and –q are placed at the base corners of an equilateral triangle whose sides are of length *a*. Show that the magnitude of the electric field intensity at the apex is the same whether one of the charges is removed or not? What is the angle between the two fields so produced?



$E = kq/r^2$; $E_1 = E_2$ *since q and r are the same for each.*

$E_y = E_1 \sin 60^0 - E_2 \sin 60 = 0$, *(since $E_1 = E_2$)*

Let E be magnitude of either E_1 or E_2, then

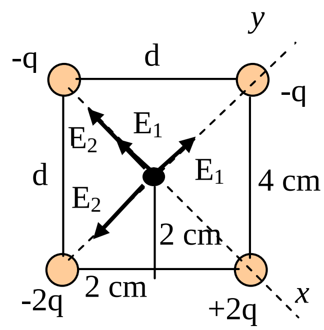
$E_x = E \sin 60^0 + E \sin 60^0 = 2E \cos 60^0 = E$

Thus, for both charges in place $E = E_1 = E_2$

The field with both charges in place is at 0^0. The field produced by –q is at -60^0 and the field produced by +q is at $+60^0$. In either case the angle is 60^0 between the fields.

*****24-39. What are the magnitude and direction of the electric field intensity at the center of the square of Fig. 24-16. Assume that q = 1 μC and that d = 4 cm. (d/2 = 2 cm).

Rotate x and y-axes 45^0 clockwise as shown to make calculating resultant easier. The distances r from each charge to center is:



$$r = \sqrt{(2\text{ cm})^2 + (2\text{ cm})^2}; \quad r = 2.83\text{ cm};$$

$$E_1 = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(1 \times 10^{-6}\text{C})}{(2.828 \times 10^{-2}\text{m})^2}; \quad E_1 = 1.125 \times 10^7 \text{ N/C}$$ *(E_1 refers to E for –q)*

$$E_2 = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-6}\text{C})}{(2.828 \times 10^{-2}\text{m})^2}; \quad E_2 = 2.25 \times 10^7 \text{ N/C},$$ *(E_2 refers to E for ±2q)*

$$E_x = -E_1 - E_2 = -1.125 \times 10^7 \text{ N/C} - 2.25 \times 10^7 \text{ N/C}; \quad E_x = -3.38 \times 10^7 \text{ N/C}$$

$$E_y = E_1 - E_2 = 1.125 \times 10^7 \text{ N/C} - 2.25 \times 10^7 \text{ N/C}; \quad E_y = -1.125 \times 10^7 \text{ N/C}$$

$$E = \sqrt{(-3.38 \times 10^7 \text{N/C})^2 + (-1.125 \times 10^7 \text{N/C})^2}; \quad E = 3.56 \times 10^7 \text{ N/C}$$

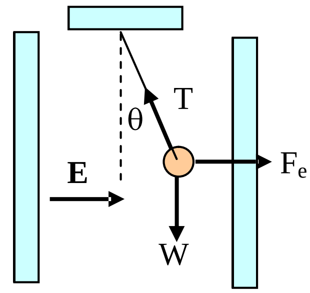
$$\tan\theta = \frac{-1.125 \times 10^7 \text{N/C}}{-3.38 \times 10^7 \text{N/C}}; \quad \theta = 18.4^0 \text{ or } 198.4^0$$ *from +x-axis*

It is better to give direction with respect to horizontal, instead of with diagonal.

Since we rotated axes 45^0 clockwise, the true angle is: $\theta = 198.4^0 - 45^0 = 153.4^0$

Ans. $E = 3.56 \times 10^7$ N, 153.4^0

*24-40. The electric field intensity between the plates in Fig. 24-17 is 4000 N/C. What is the magnitude of the charge on the suspended pith ball whose mass is 3 mg? ($\theta = 30^0$)



$W = mg$; E = 4000 N/C; m = 3 mg = 3 x 10^{-6} kg

$\Sigma F_x = 0$ *and* $\Sigma F_y = 0$ *(right = left; up = down)*

T sin 60^0 = (3 x 10^{-6} kg)(9.8 m/s^2); T = 3.395 x 10^{-5} N

F_e = T cos 60^0 = (3.395 x 10^{-5} N)(0.500) = 1.70 x 10^{-5} N

$$E = \frac{F_e}{q}; \quad q = \frac{F_e}{E} = \frac{1.70 \text{ x } 10^{-5}\text{N}}{4000 \text{ N/C}}; \quad q = 4.24 \text{ x } 10^{-9} \text{ C}; \quad \boxed{q = 4.24 \text{ nC}}$$

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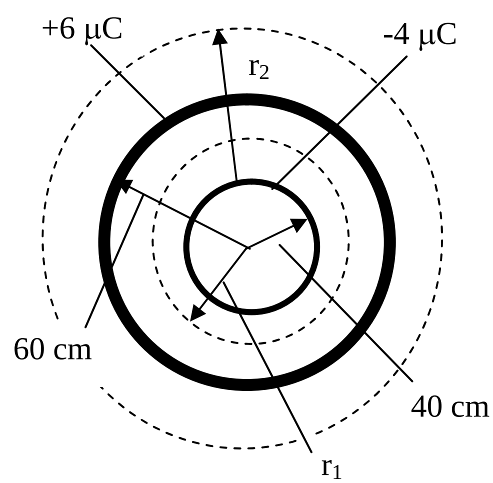
*24-41. Two concentric spheres have radii of 20 cm and 50 cm. The inner sphere has a negative charge of –4 μC and the outer sphere has a positive charge of +6 μC. Use Gauss's law to find the electric field intensity at distances of 40 cm and 60 cm from the center of the spheres. *Draw concentric gaussian spheres.*



$$\Sigma \varepsilon_0 AE = \Sigma q; \quad \varepsilon_0 (4\pi r_2^2)E = -4\ \mu\text{C} + 6\ \mu\text{C}$$

First find field at 60 cm from center:

$$E = \frac{q_{net}}{4\pi\varepsilon_0 r^2} = \frac{+2 \text{ x } 10^{-6}\text{C}}{4\pi(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.60 \text{ m})^2};$$

$$\boxed{E = 5.00 \text{ x } 10^4 \text{ N/C}, \textit{ radially outward}}$$

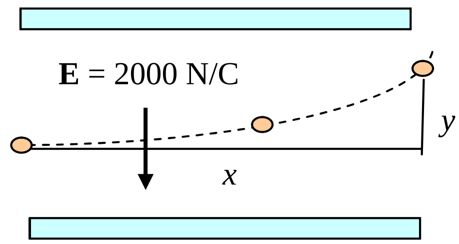
Now for field at 40 cm, only enclosed charge matters.

$$E = \frac{q_{net}}{4\pi\varepsilon_0 r^2} = \frac{-4 \text{ x } 10^{-6}\text{C}}{4\pi(8.85 \text{ x } 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.40 \text{ m})^2};$$

$$\boxed{E = 2.25 \text{ x } 10^5 \text{ N/C}, \textit{ radially inward}}$$

*24-42. The electric field intensity between the two plates in Fig. 24-4 is 2000 N/C. The length of the plates is 4 cm, and their separation is 1 cm. An electron is projected into the field from the left with horizontal velocity of 2 x 10^7 m/s. What is the upward deflection of the electron at the instant it leaves the plates?

We may neglect the weight of the electron.



$$F = qE = ma_y; \quad a_y = \frac{qE}{g}; \quad x = v_0 t$$

$$y = ½ a_y t^2 \text{ and } t = \frac{x}{v_0}; \quad t^2 = \frac{x^2}{v_0^2}$$

$$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x^2}{v_0^2}\right) = \frac{1}{2}\left[\frac{(1.6 \times 10^{-19}\text{C})(2000 \text{ N/C})(0.04 \text{ m})^2}{(9.11 \times 10^{-31}\text{kg})(2 \times 10^7 \text{m/s})^2}\right]$$

y = 0.0704 cm *or* y = 0.70 mm