Chapter 23. The Electric Force

Coulomb's Law

23-1. Two balls each having a charge of 3 μC are separated by 20 mm. What is the force of repulsion between them?

$$F = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(3 \text{ x } 10^{-6}\text{C})(3 \text{ x } 10^{-6}\text{C})}{(20 \text{ x } 10^{-3}\text{m})^2}; \quad \boxed{F = 202 \text{ N}}$$

23-2. Two point charges of -3 and +4 μC are 12 mm apart in a vacuum. What is the electrostatic force between them?

$$F = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(-3 \text{ x } 10^{-6}\text{C})(4 \text{ x } 10^{-6}\text{C})}{(12 \text{ x } 10^{-3}\text{m})^2}; \quad \boxed{F = 750 \text{ N, attraction}}$$

23-3. An alpha particle consists of two protons (q_e = 1.6 x 10^{-19} C) and two neutrons (no charge). What is the repulsive force between two alpha particles separated by 2 nm?

$$q_\alpha = 2(1.6 \text{ x } 10^{-19} \text{ C}) = 3.2 \text{ x } 10^{-19} \text{ C}$$

$$F = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(3.2 \text{ x } 10^{-19}\text{C})(3.2 \text{ x } 10^{-19}\text{C})}{(2.00 \text{ x } 10^{-9}\text{m})^2}; \quad \boxed{F = 2.30 \text{ x } 10^{-10} \text{ N}}$$

23-4. Assume that the radius of the electron's orbit around the proton in a hydrogen atom is approximately 5.2 x 10^{-11} m. What is the electrostatic force of attraction?

$$F = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \text{ x } 10^{-19}\text{C})(-1.6 \text{ x } 10^{-19}\text{C})}{(5.2 \text{ x } 10^{-11}\text{m})^2}; \quad \boxed{F = 8.52 \text{ x } 10^{-8} \text{ N}}$$

23-5. What is the separation of two -4 μC charges if the force of repulsion between them is 200 N?

$$r = \sqrt{\frac{kqq'}{F}} = \sqrt{\frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(-4 \text{ x } 10^{-6}\text{C})^2}{200 \text{ N}}}; \quad \boxed{r = 26.8 \text{ mm}}$$

23-6. Two identical charges separated by 30 mm experience a repulsive force of 980 N. What is the magnitude of each charge?

$$F = \frac{kq^2}{r^2}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(980\text{ N})(0.030\text{ m})^2}{9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2}}; \quad \boxed{q = 9.90\ \mu\text{C}}$$

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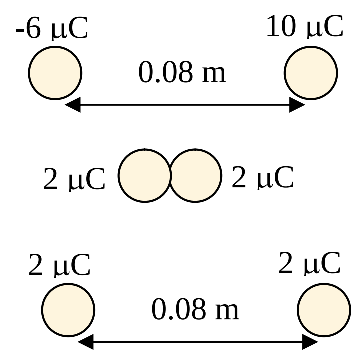
*23-7. A 10 μC charge and a -6 μC charge are separated by 40 mm. What is the force between them. The spheres are placed in contact for a few moments and then separated again by 40 mm. What is the new force? Is it attractive or repulsive?

$$F = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(-6 \times 10^{-6}\text{C})(10 \times 10^{-6}\text{C})}{(40 \times 10^{-3}\text{m})^2}; \quad \boxed{F = 338\text{ N, attraction}}$$



When spheres touch, 6 μC *of charge are neutralized, leaving* 4 μC *to be shared by two spheres, or* +2 μC *on each sphere. Now they are again separated.*

$$F = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-6}\text{C})(2 \times 10^{-6}\text{C})}{(0.080\text{ m})^2};$$

$$\boxed{F = 5.62\text{ N, repulsion}}$$

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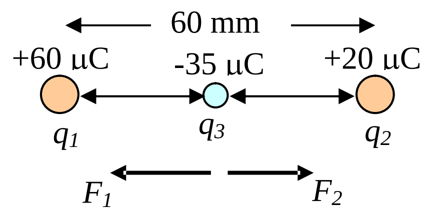
*23-8. Two point charges initially attract each other with a force of 600 N. If their separation is reduced to one-third of its original distance, what is the new force of attraction?

$$F \propto \frac{1}{r^2}; \quad F_1 r_1^2 = F_2 r_2^2; \quad F_2 = F_1\left(\frac{r_1^2}{r_2^2}\right) = F_1\left(\frac{r_1}{r_2}\right)^2; \quad r_1 = 3\, r_2$$

$$F_2 = F_1\left(\frac{3r_2}{r_2}\right)^2 = 9F_1; \quad \boxed{F_2 = 5400\text{ N}}$$

The Resultant Electrostatic Force

23-9. A +60 μC charge is placed 60 mm to the left of a +20 μC charge. What is the resultant force on a -35 μC charge placed midway between the two charges?



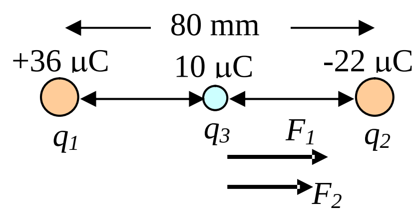
$$F_{13} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(60 \times 10^{-6}\text{C})(35 \times 10^{-6}\text{C})}{(30 \times 10^{-3}\text{m})^2}$$

F_{13} = 2.10 x 10^4 N, *directed to the* *left*

$$F_{23} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(20 \times 10^{-6}\text{C})(35 \times 10^{-6}\text{C})}{(30 \times 10^{-3}\text{m})^2};$$ F_{13} = 2.10 x 10^4 N, *directed to* *right.*

$F_R = F_{13} + F_{23}$ = (-2.10 x 10^4 N) + (0.700 x 10^4 N); **F_R = -1.40 x 10^4 N, left.**

23-10. A point charge of +36 μC is placed 80 mm to the left of a second point charge of -22 μC. What force is exerted on third charge of +10 μC located at the midpoint?



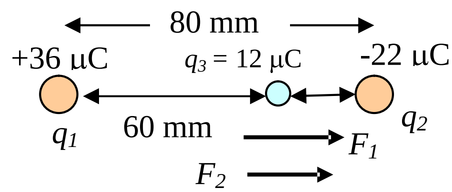
$$F_{13} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-6}\text{C})(10 \times 10^{-6}\text{C})}{(40 \times 10^{-3}\text{m})^2}$$

F_{13} = 2025 N, *directed to the* *right*

$$F_{23} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(22 \times 10^{-6}\text{C})(10 \times 10^{-6}\text{C})}{(40 \times 10^{-3}\text{m})^2};$$ F_{13} = 1238 N, *directed to* *right.*

$F_R = F_{13} + F_{23}$ = 2025 N + 1238 N; **F_R = 3260 N, *left.***

23-11. For Problem 23-10, what is the resultant force on a third charge of +12 μC placed between the other charges and located 60 mm from the +36 μC charge?



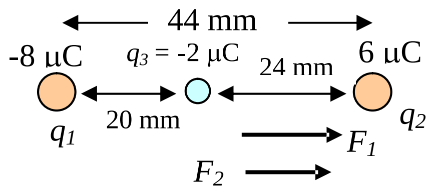
$$F_{13} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-6}\text{C})(12 \times 10^{-6}\text{C}}{(60 \times 10^{-3}\text{m})^2}$$

$$F_{23} = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(22 \times 10^{-6}\text{C})(12 \times 10^{-6}\text{C})}{(20 \times 10^{-3}\text{m})^2};$$

Both to right, so $F_R = F_{13} + F_{23}$ = 1080 N + 5940 N; **F = 7020 N, *rightward.***

23-12. A +6 μC charge is 44 mm to the right of a -8 μC charge. What is the resultant force on a -2 μC charge that is 20 mm to the right of the -8 μC charge?

$$F_{13} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(8 \text{ x } 10^{-6}\text{C})(2 \text{ x } 10^{-6}\text{C})}{(20 \text{ x } 10^{-3}\text{m})^2}$$



$$F_{23} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2 \text{ x } 10^{-6}\text{C})(6 \text{ x } 10^{-6}\text{C})}{(24 \text{ x } 10^{-3}\text{m})^2};$$

Both to right, so $F_R = F_{13} + F_{23}$ = 360 N + 187.5 N; **F = 548 N, *rightward***

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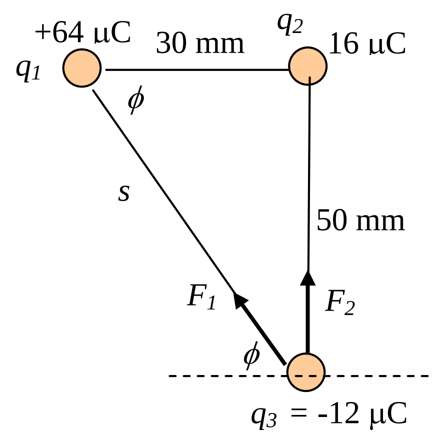
*23-13. A 64-μC charge is locate 30 cm to the left of a 16-μC charge. What is the resultant force on a -12 μC charge positioned exactly 50 mm below the 16 μC charge?

$$s = \sqrt{(30 \text{ mm})^2 + (50 \text{ mm})^2} = 58.3 \text{ mm}$$

$$\tan\phi = \frac{50 \text{ mm}}{30 \text{ mm}}; \quad \phi = 59.0^0$$



$$F_{13} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(64 \text{ x } 10^{-6}\text{C})(12 \text{ x } 10^{-6}\text{C})}{(58.3 \text{ x } 10^{-3}\text{m})^2}$$

F_{13} = 2033 N, 59.0^0 N of W

$$F_{23} = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(16 \text{ x } 10^{-6}\text{C})(12 \text{ x } 10^{-6}\text{C})}{(50 \text{ x } 10^{-3}\text{m})^2} = 691 \text{ N, upward.}$$

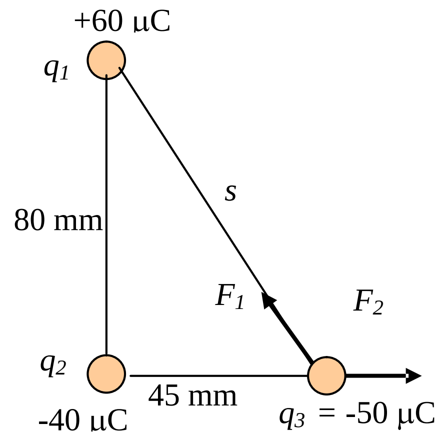
$F_x = 0 - F_{13} \cos 59.0^0 = -(2033 \text{ N}) \cos 59^0$; $F_x = -1047$ N

$F_y = F_{23} + F_{13} \sin 59.0^0 = 691 \text{ N} + (2033 \text{ N}) \sin 59^0$; $F_y = 2434$ N

$$F = \sqrt{(-1047 \text{ N})^2 + (2434 \text{ N})^2} = 2650 \text{ N}; \quad \tan\theta = \frac{2434 \text{ N}}{-1047 \text{ N}}; \quad \theta = 66.7^0 \text{ N of W.}$$

Resultant force: F_R = 2650 N, 66.7^0 N of W (or 113.3^0)

*23-14. A charge of +60 nC is located 80 mm above a -40-nC charge. What is the resultant force on a -50-nC charge located 45 mm horizontally to the right of the -40-nC charge?



$$s = \sqrt{(45\text{ mm})^2 + (80\text{ mm})^2} = 91.8\text{ mm}$$

$$\tan\phi = \frac{80\text{ mm}}{45\text{ mm}}; \quad \phi = 60.64^0$$

$$F_{13} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(60\text{ x }10^{-6}\text{C})(50\text{ x }10^{-6}\text{C})}{(91.8\text{ x }10^{-3}\text{m})^2}$$

F_{13} = 2564 N, 60.64^0 N of W

$$F_{23} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(40\text{ x }10^{-6}\text{C})(50\text{ x }10^{-6}\text{C})}{(45\text{ x }10^{-3}\text{m})^2} = 8889\text{ N, } \textit{rightward}.$$

$F_x = -F_{13}\cos 60.64^0 + F_{23} = -(2564\text{ N})\cos 60.64^0 + 8889\text{ N}$; $F_x = 7632$ N

$F_y = +F_{13}\sin 60.64^0 + 0 = (2564\text{ N})\sin 60.64^0$; $F_y = 2235$ N

$$F = \sqrt{(7632\text{ N})^2 + (2235\text{ N})^2}; \quad \tan\theta = \frac{2235\text{ N}}{7632\text{ N}};$$

F_R = 7950 N, θ = 16.3^0 N of E.

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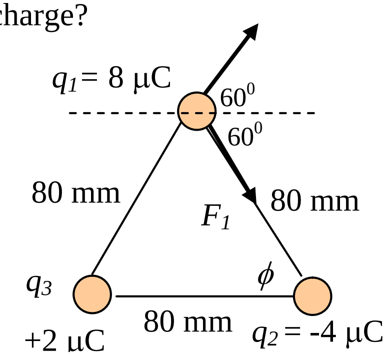
*23-15. Three point charges q_1 = + 8 μC, q_2 = -4 μC, and q_3 = +2 μC are placed at the corners of an equilateral triangle, 80 mm on each side as described by Fig. 23-15. What are the magnitude and direction of the resultant force on the +8 μC charge?



$$F_{21} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(4\text{ x }10^{-6}\text{C})(8\text{ x }10^{-6}\text{C})}{(80\text{ x }10^{-3}\text{m})^2}$$

F_{21} = 45.0 N, 60^0 S of E

$$F_{31} = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2\text{ x }10^{-6}\text{C})(8\text{ x }10^{-6}\text{C})}{(80\text{ x }10^{-3}\text{m})^2}$$

F_{31} = 22.5 N, 60^0 N of E; $F_x = (22.5\text{ N})\cos 60^0 + (45\text{ N})\cos 60^0 = 33.8$ N

$F_y = (22.5\text{ N})\sin 60^0 - (45\text{ N})\sin 60^0 = -19.5$ N

*23-15. (Cont.) $F = \sqrt{(33.8\text{ N})^2 + (-19.5\text{ N})^2} = 39.0\text{ N}; \quad \tan\theta = \frac{-19.5\text{ N}}{33.8\text{ N}}; \quad \theta = -30^0$

Resultant electric force: $F_R = 39.0\text{ N}, \quad \theta = 330.0^0$

Challenge Problems

23-16. What should be the separation of two +5-uC charges so that the force of repulsion is 4 N?

$$r = \sqrt{\frac{kqq'}{F}} = \sqrt{\frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(5\text{ x }10^{-6}\text{C})^2}{4.00\text{ N}}}; \quad \boxed{r = 23.7\text{ cm}}$$

23-17. The repulsive force between two pith balls is found to be 60 μN. If each pith ball carries a charge of 8 nC, what is their separation?

$$r = \sqrt{\frac{kqq'}{F}} = \sqrt{\frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(8\text{ x }10^{-9}\text{C})^2}{60\text{ x }10^{-6}\text{ N}}}; \quad \boxed{r = 98.0\text{ mm}}$$

23-18. Two identical unknown charges experience a mutual repulsive force of 48 N when separated by 60 mm. What is the magnitude of each charge?

$$F = \frac{kq^2}{r^2}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(48\text{ N})(0.060\text{ m})^2}{9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2}}; \quad \boxed{q = 4.38\ \mu\text{C}}$$

23-19. One object contains an excess of 5 x 10^{14} electrons and another has a deficiency of 4 x 10^{14} electrons. What is the force each exerts on the other if the objects are 30 mm apart? Is it attraction or repulsion? (1*e* = 1.6 x 10^{-19} C, *excess* = -, *deficiency* = +.)

$q_1 = (5\text{ x }10^{14}\ e)(1.6\text{ x }10^{-19}\text{ C}/e) = -80\ \mu\text{C}; \quad q_2 = (4\text{ x }10^{14}\ e)(1.6\text{ x }10^{-19}\text{ C}/e) = +64\ \mu\text{C}$

$$F = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(80\text{ x }10^{-6}\text{C})(64\text{ x }10^{-6}\text{C})}{(30\text{ x }10^{-3}\text{m})^2}; \quad \boxed{F = 5.12\text{ x }10^4\text{ N, attraction}}$$

23-20. If it were possible to put 1 C of charge on each of two spheres separated by a distance of 1 m, what would be the repulsive force in newtons.

$$F = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(1\text{ C})(1\text{ C})}{(1\text{ m})^2}; \quad \boxed{F = 9 \times 10^9 \text{ N !}}$$

The coulomb is a very large unit for electrostatics applications.

23-21. How many electrons must be placed on each of two spheres separated by 4 mm in order to produce a repulsive force of one 400 N?

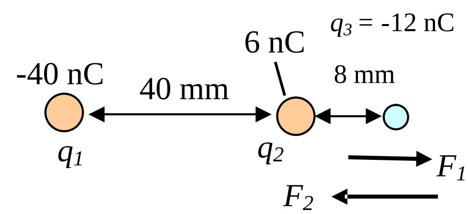
$$F = \frac{kq^2}{r^2}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(400\text{ N})(0.004\text{ m})^2}{9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2}}; \quad q = 843\text{ nC}$$

$$q = 843 \times 10^{-9}\text{C}\left(\frac{1\text{ e}}{1.6 \times 10^{-19}\text{C}}\right); \quad \boxed{q = 5.27 \times 10^{12} \text{ electrons}}$$

23-22. A –40-nC charge is placed 40 mm to the left of a +6-nC charge. What is the resultant force on a –12-nC charge place 8 mm to the right of the +6-nC charge?

$$F_1 = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(40 \times 10^{-9}\text{C})(12 \times 10^{-9}\text{C})}{(48 \times 10^{-3}\text{m})^2}$$

$$F_2 = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(6 \times 10^{-9}\text{C})(12 \times 10^{-9}\text{C})}{(8 \times 10^{-3}\text{m})^2};$$

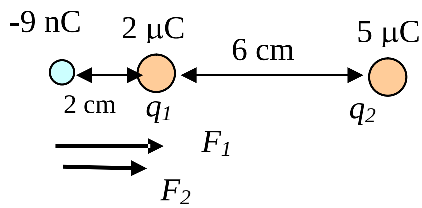


Both to right, so $F_R = F_1 + F_2$ = 1.88 mN – 10.1 mN; $\boxed{F = -8.25 \text{ mN, } \textit{leftward}}$

23-23. A 5-μC charge is placed 6 cm to the right of a 2-μC charge. What is the resultant force on a –9 nC charge placed 2 cm to the left of the 2-μC charge?

$$F_1 = \frac{(9 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(9 \times 10^{-9}\text{C})(2 \times 10^{-6}\text{C})}{(2 \times 10^{-2}\text{m})^2}$$

F_1 = +405 mN, *to right*



$$F_2 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(5 \text{ x } 10^{-6}\text{C})(9 \text{ x } 10^{-9}\text{C})}{(8 \text{ x } 10^{-2}\text{m})^2}; \quad F_2 = +63.3 \text{ mN, } \textit{to right}$$

Resultant force: F_R = 405 mN + 63.3 mN; **F_R = 468 mN**

23-24. An equal number of electrons is placed on two metal spheres 3.0 cm apart in air. How many electrons are on each sphere if the resultant force is 4500 N?

$$F = \frac{kq^2}{r^2}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(4500 \text{ N})(0.03 \text{ m})^2}{9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2}}; \quad q = 21.2\ \mu\text{C}$$

$$q = 21.2 \text{ x } 10^{-6}\text{C}\left(\frac{1 \text{ e}}{1.6 \text{ x } 10^{-19}\text{C}}\right); \quad \boxed{q = 1.33 \text{ x } 10^{14} \text{ electrons}}$$

23-25. A 4-nC charge is placed on a 4-g sphere that is free to move. A fixed 10-μC point charge is 4 cm away. What is the initial acceleration of the 4-μC charge?

$$F_2 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(4 \text{ x } 10^{-9}\text{C})(10 \text{ x } 10^{-6}\text{C})}{(4 \text{ x } 10^{-2}\text{m})^2}; \quad F_2 = 225 \text{ mN}$$

$$a = \frac{F}{m} = \frac{0.225 \text{ N}}{0.004 \text{ kg}} \quad \boxed{a = 56.2 \text{ m/s}^2}$$

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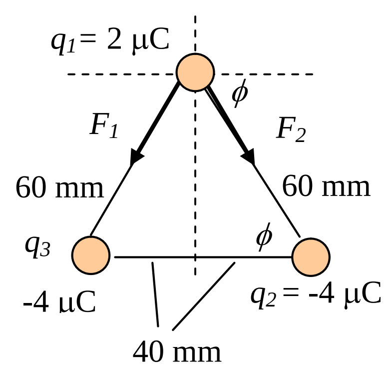
*23-26. What is the resultant force on a +2 μC charge an equal distance of 60 mm from each of two –4-μC charges that are 80 mm apart in air?



$$\tan\phi = \frac{40 \text{ mm}}{60 \text{ mm}}; \quad \phi = 48.2^0$$

$$F_2 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(4 \text{ x } 10^{-6}\text{C})(2 \text{ x } 10^{-6}\text{C})}{(60 \text{ x } 10^{-3}\text{m})^2}$$

F_2 = 20.0 N, 48.2^0 S of E

$$F_1 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2 \text{ x } 10^{-6}\text{C})(4 \text{ x } 10^{-6}\text{C})}{(60 \text{ x } 10^{-3}\text{m})^2}; \quad F_1 = 20.0 \text{ N, } 48.2^0 \text{ S of W}$$

*23-26. (Cont.) $F_x = (20\text{ N})\cos 48.2^0 + (20\text{ N})\cos 48.2^0 = 13.33\text{ N} - 13.33\text{ N};\quad F_x = 0$

$F_y = (20\text{ N})\sin 48.2^0 + (20\text{ N})\sin 48.2^0 = 14.9\text{ N} + 14.9\text{ N};\quad F_y = -29.8\text{ N}$

Resultant force: $F_R = 29.8$ N, *downward*

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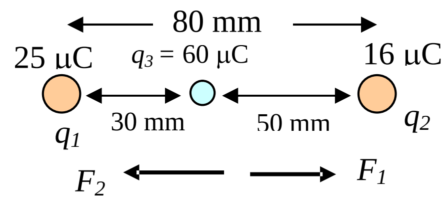
*23-27. Two charges of +25 and +16 μC are 80 mm apart. A third charge of +60 μC is placed between the other charges 30 mm from the +25 μC charge. Find the resultant force on the third charge?



$$F_1 = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(60\text{ x }10^{-6}\text{C})(25\text{ x }10^{-6}\text{C})}{(30\text{ x }10^{-3}\text{m})^2}$$

$F_1 = 15$ kN, *directed to the right*

$$F_2 = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(60\text{ x }10^{-6}\text{C})(16\text{ x }10^{-6}\text{C})}{(50\text{ x }10^{-3}\text{m})^2};\quad F_{13} = 3.46\text{ kN, directed to left.}$$

$F_R = F_1 + F_2 = 15\text{ kN} + 3.46\text{ kN});$ $F_R = 11.5$ kN, *right.*

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*23-28. A 0.02-g pith ball is suspended freely. The ball is given a charge of +20 μC and placed 0.6 m from a charge of +50 μC. What will be the initial acceleration of the pith ball?

$$F = \frac{(9\text{ x }10^9\text{N}\cdot\text{m}^2/\text{C}^2)(20\text{ x }10^{-6}\text{C})(50\text{ x }10^{-6}\text{C})}{(0.600\text{ m})^2};\quad F = 25.0\text{ N}$$

$$a = \frac{F}{m} = \frac{25.0\text{ N}}{2\text{ x }10^{-5}\text{kg}};\quad a = 1.25\text{ x }10^6\text{ m/s}$$

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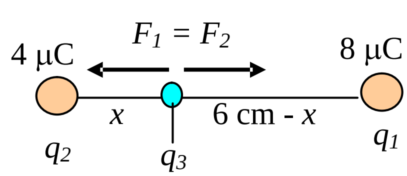
*23-29. A 4 μC charge is located 6 cm from an 8 μC charge. At what point on a line joining the two charges will the resultant force be zero?



$$F_1 = F_2;\qquad \frac{kq_3q_1}{x^2} = \frac{kq_3q_2}{(6-x)^2}$$

$$F_1 = F_2; \quad \frac{q_1}{x^2} = \frac{q_2}{(6-x)^2}; \quad x^2 = \left(\frac{q_1}{q_2}\right)(6-x)^2$$

Take square root of both sides: $x = \sqrt{\frac{q_1}{q_2}}(6 - x) = \sqrt{\frac{4\ \mu C}{8\ \mu C}}(6 - x); \quad x = 0.707(6 - x)$

Solving for x, we obtain: **x = 2.49 cm *from* 4-μC *charge***

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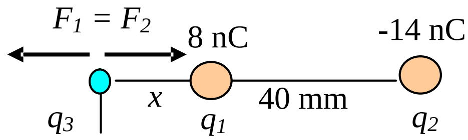
*23-30. A charge of +8 nC is placed 40 mm to the left of a –14 nC charge. Where should a third charge be placed if it is to experience a zero resultant force?

Considering the sign of the charges and their magnitudes, the charge must be to the left of the 8 nC charge as shown.



$$F_1 = F_2; \quad \frac{kq_3q_1}{x^2} = \frac{kq_3q_2}{(40+x)^2} \qquad F_1 = F_2; \quad \frac{q_1}{x^2} = \frac{q_2}{(40+x)^2}; \quad x^2 = \left(\frac{q_1}{q_2}\right)(40+x)^2$$

Take square root of both sides:

$$x = \sqrt{\frac{q_1}{q_2}}(40 + x) = \sqrt{\frac{8\text{ nC}}{14\text{ nC}}}(40 + x); \quad x = 0.756(40 + x)$$

Solving for x, we obtain: **x = 124 mm *left of* 8 nC *charge.***

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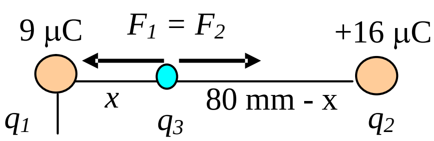
*23-31. A +16-μC charge is 80 mm to the right of a +9 μC. Where should a third charge be placed so that the resultant force is zero?



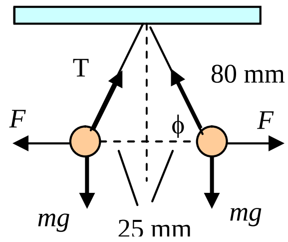
$$F_1 = F_2; \quad \frac{kq_3q_1}{x^2} = \frac{kq_3q_2}{(80-x)^2}$$

$$F_1 = F_2; \quad \frac{q_1}{x^2} = \frac{q_2}{(80-x)^2}; \quad x^2 = \left(\frac{q_1}{q_2}\right)(80-x)^2$$

Take square root of both sides: $x = \sqrt{\frac{q_1}{q_2}}(80 - x) = \sqrt{\frac{9\ \mu C}{16\ \mu C}}(80 - x); \quad x = 0.750(80 - x)$

Solving for x, we obtain: **x = 34.3 mm *from* 9-μC *charge***

*23-32. Two 3-g spheres are suspended from a common point with two 80 mm light silk threads of negligible mass. What charge must be placed on each sphere if their final positions are 50 mm apart?



$$\cos\phi = \frac{25\text{ mm}}{80\text{ mm}}; \quad \phi = 71.8^0 \quad T_y = T\sin\phi = mg$$

$$T = \frac{mg}{\sin\phi} = \frac{(3 \times 10^{-3}\text{kg})(9.8\text{ m/s}^2)}{\sin 71.8^0}; \quad T = 30.9\text{ mN}$$

$$T_x = T\cos 71.8^0 = (30.9\text{ mN})\cos 71.8^0; \quad T_x = 9.67\text{ mN}; \quad \Sigma F_x = 0: \quad F = T_x$$

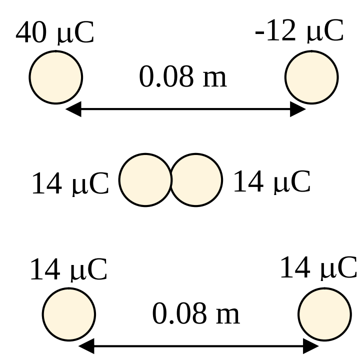
$$F = \frac{kq^2}{r^2} = 9.67\text{ mN}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(9.67 \times 10^{-3}\text{N})(50 \times 10^{-3}\text{m})^2}{9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2}}; \quad \boxed{q = 51.8\text{ nC}}$$

Critical Thinking Questions

*23-33. A small metal sphere is given a charge of +40 μC, and a second sphere 8 cm away is given a charge of –12 μC. What is the force of attraction between them? If the two spheres are allowed to touch and are then again placed 8 cm apart, what new electric force exists between them? Is it attraction or repulsion?

$$F = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-6}\text{C})(40 \times 10^{-6}\text{C})}{(0.080\text{ m})^2}; \quad \boxed{F = 675\text{ N, attraction}}$$

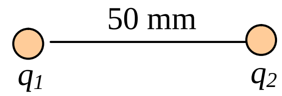
When spheres touch, 6 μC of charge are neutralized, leaving 28 μC to be shared by two spheres, or +14 μC *on each sphere. Now they are again separated:*



$$F = \frac{(9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(14 \times 10^{-6}\text{C})(14 \times 10^{-6}\text{C})}{(0.080\text{ m})^2};$$

$$\boxed{F = 276\text{ N, repulsion}}$$

*23-34. The total charge on two metal spheres 50 mm apart is 80 μC. If they repel each other with a force of 800 N, what is the charge on each sphere?



$q_1 + q_2 = 80\ \mu C; \quad q_2 = 80\ \mu C - q_1$

$$F = \frac{kq_1q_2}{r^2} = \frac{kq_1(80\mu C - q_1)}{r^2}; \quad F = \frac{kq_1(80\mu C) - kq_1^2}{r^2}; \quad \frac{Fr^2}{k} = (80\ \mu C)q_1 - q_1^2$$

$$\frac{(800\text{ N})(50 \text{ x } 10^{-3}\text{m})^2}{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)} = (80\ \mu C)q_1 - q_1^2; \quad q_1^2 - (80 \text{ x } 10^{-6}\text{ C})q_1 + 222 \text{ x } 10^{-12}\text{ C}^2 = 0$$

Solve the quadratic equation with $a = 1$, $b = -80 \text{ x } 10^{-6}$, *and* $c = 222 \text{ x } 10^{-12}$

$q_1 = 77.1\ \mu C$ and $q_1 = 2.89\ \mu C$

Now $q_2 = 80\ \mu C - q_1$ *yields the following :* $q_2 = 2.89\ \mu C$ and $q_2 = 77.1\ \mu C$

Thus, one charge is 77.1 μC and the other is 2.89 μC

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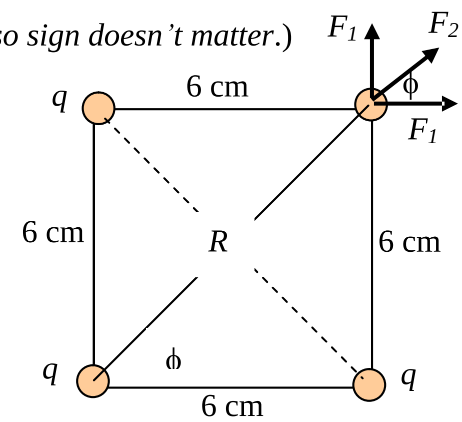
*23-35. Four small spheres are each given charges of q = +20 μC and placed at the corners of a square with sides of length 6 cm. Show that the resultant force on each charge has a magnitude equal to 1914 N. What is the direction of the force? What will change if the charges are each q = -20 μC? (*Like charges repel, so sign doesn't matter*.)



$$R = \sqrt{(6\text{ cm})^2 + (6\text{ cm})^2} = 8.485\text{ cm}$$

$$F_1 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(20 \text{ x } 10^{-6}\text{C})(20 \text{ x } 10^{-6}\text{C})}{(6 \text{ x } 10^{-2}\text{m})^2}$$

$$F_2 = \frac{(9 \text{ x } 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(20 \text{ x } 10^{-6}\text{C})(20 \text{ x } 10^{-6}\text{C})}{(8.485 \text{ x } 10^{-2}\text{m})^2}$$

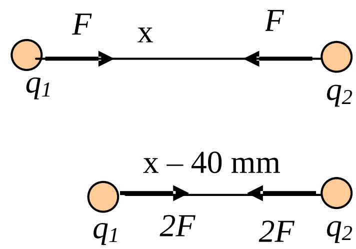
F_1 = 1000 N; F_2 = 500 N; *For a square, the angle* $\phi = 45^0$

$F_x = (1000\text{ N}) + (500\text{ N})\cos 45^0 + 0 = 1354\text{ N}$; $F_y = 1000\text{ N} + (500\text{ N})\sin 45^0 = 1354\text{ N}$

$$F = \sqrt{(1354\text{ N})^2 + (1354\text{ N})^2};$$

F = 1914 N, 45^0 *away from center.*

*23-36. Two charges q_1 and q_2 are separated by a distance r. They experience a force F at this distance. If the initial separation is decreased by only 40 mm, the force between the two charges is doubled. What was the initial separation?

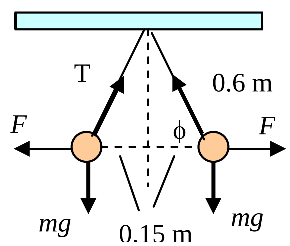


$$\frac{2kq_1q_2}{x^2} = \frac{kq_1q_2}{(x-40\text{ mm})^2}; \quad \frac{2}{x^2} = \frac{1}{(x-40\text{ mm})^2}$$

$x^2 = 2(x – 40\text{ mm})^2$ T*ake square root of both sides:*

$x = 1.414(x – 40\text{ mm})$; **x = 137 mm**

*23-37. Two 8-g pith balls are suspended from silk threads 60 cm long and attached to a common point. When the spheres are given equal amounts of negative charge, the balls come to rest 30 cm apart. Calculate the magnitude of the charge on each pith ball.



$$\cos\phi = \frac{0.15\text{ m}}{0.60\text{ m}}; \quad \phi = 75.5^0 \quad T_y = T\sin\phi = mg$$

$$T = \frac{mg}{\sin\phi} = \frac{(8 \times 10^{-3}\text{kg})(9.8\text{ m/s}^2)}{\sin 75.5^0}; \quad T = 81.0\text{ mN}$$

$T_x = T\cos 71.8^0 = (81.0\text{ mN})\cos 75.5^0$; $T_x = 20.25\text{ mN}$; $\Sigma F_x = 0$: $F = T_x$

$$F = \frac{kq^2}{r^2} = 20.25\text{ mN}; \quad q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(2.025 \times 10^{-4}\text{N})(0.30\text{ m})^2}{9 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2}};$$

q = -450 nC